



CK-12 Texas Instruments Geometry

Teacher's Edition



Texas Instruments Geometry Teacher's Edition

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flexbook
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1	TE Introduction to Geometry - TI	1
1.1	Geometry TI Resources Flexbook	2
2	TE Basics of Geometry - TI	3
2.1	Midpoints in the Coordinate Plane	4
2.2	Vertical and Adjacent Angles	7
3	TE Reasoning and Proof - TI	10
3.1	Conditional Statements	11
4	TE Parallel and Perpendicular Lines - TI	15
4.1	Parallel Lines cut by a Transversal	16
4.2	Transversals	17
4.3	Perpendicular Slopes	20
5	TE Congruent Triangles - TI	21
5.1	Interior and Exterior Angles of a Triangle	22
5.2	Congruent Triangles	26
5.3	Triangle Sides and Angles	31
6	TE Relationships within Triangles - TI	34
6.1	Perpendicular Bisector	35
6.2	Hanging with the Incenter	37
6.3	Balancing Point	40
6.4	Hey Ortho! What's your Altitude?	43
7	TE Quadrilaterals - TI	46
7.1	Properties of Parallelograms	47
7.2	Properties of Trapezoids and Isosceles Trapezoids	50
7.3	Properties of Rhombi, Kites and Trapezoids	51
8	TE Similarity - TI	54
8.1	Constructing Similar Triangles	55

8.2	Side-Splitter Theorem	58
8.3	Perspective Drawing	60
9	TE Right Triangle Trigonometry - TI	64
9.1	The Pythagorean Theorem	65
9.2	Investigating Special Triangles	70
9.3	Ratios of Right Triangles	73
10	TE Circles - TI	77
10.1	Chords and Circles	78
10.2	Inscribed Angles Theorem	80
10.3	Circle Product Theorems	83
11	TE Perimeter and Area - TI	86
11.1	Diameter and Circumference of a Circle	87
11.2	From the Center of the Polygon	89
12	TE Surface Area and Volume - TI	92
12.1	Surface Area of a Cylinder	93
13	TE Transformations - TI	95
13.1	Transformations with Lists	96
13.2	Reflections and Rotations	100
13.3	Perspective Drawing	102

CHAPTER

1

TE Introduction to Geometry - TI

CHAPTER OUTLINE

1.1 GEOMETRY TI RESOURCES FLEXBOOK

1.1 Geometry TI Resources Flexbook

Teacher's Edition

Introduction

This flexbook contains Texas Instruments (TI) Resources for the TI-83, TI-83 Plus, TI-84, and TI-84 SE. All the activities in this flexbook supplement the lessons in the student edition. Teachers may need to download programs from www.timath.com that will implement or assist in the activities. Each activity included is designed to help the teacher with each activity in the TI Resources Flexbook, Student Edition. All activities are listed in the same order as the Student Edition.

There are also corresponding links in the 1st Edition of Geometry, 2nd Edition, and Basic Geometry.

- Geometry, first edition: <http://www.ck12.org/flexr/flexbook/805>
- Geometry, second edition: <http://www.ck12.org/flexr/flexbook/3461>
- Basic Geometry: <http://www.ck12.org/flexr/flexbook/3129>

Any activity that requires a calculator file or application, go to <http://www.education.ti.com/calculators/downloads> and type the name of the activity or program in the search box.

CHAPTER

2

TE Basics of Geometry - TI

CHAPTER OUTLINE

2.1 MIDPOINTS IN THE COORDINATE PLANE

2.2 VERTICAL AND ADJACENT ANGLES

2.1 Midpoints in the Coordinate Plane

This activity is intended to supplement Geometry, Chapter 1, Lesson 4.

ID: 8614

Time required: 40 minutes

Topic: Points, Lines #38; Planes

- *Given the coordinates of the ends of a line segment, write the coordinates of its midpoint.*

Activity Overview

In this activity, students will explore midpoints in the coordinate plane. Beginning with horizontal or vertical segments, students will show the coordinates of the endpoints and make a conjecture about the coordinates of the midpoint. This conclusion is extended to other segments in the coordinate plane.

Teacher Preparation

- *This activity is designed to be used in a high school or middle school geometry classroom.*
- *The screenshots on pages 1–3 demonstrate expected student results.*
- *The Coordinate Midpoint formula for the midpoint of (x_1, y_1) and (x_2, y_2) is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. This can also be expressed as “The coordinates of the midpoint of a line segment are the averages of the coordinates of the endpoints.”*

Classroom Management

- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet provides a place for students to record their observations.*
- *Depending on student skill level, you may wish to use points with integer coordinates, or only positive values.*
- **Note:** *The coordinates can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To ensure that a point is actually at an integer value rather than a rounded decimal value, do the following:*
 1. Move the cursor over the coordinate value so it is highlighted.
 2. Press + to display additional decimal digits or - to hide digits.
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*

Associated Materials

- Student Worksheet: Midpoints in the Coordinate Plane <http://www.ck12.org/flexr/chapter/9686>
- Cabri Jr. Application

Problem 1 – Midpoints of Horizontal or Vertical Segments

Step 1:

- Students should open a new Cabri Jr. file. If the axes are not currently showing, they should select **Hide/Show #62; Axes**.

They will construct a horizontal segment in the first quadrant using the **Segment** tool.

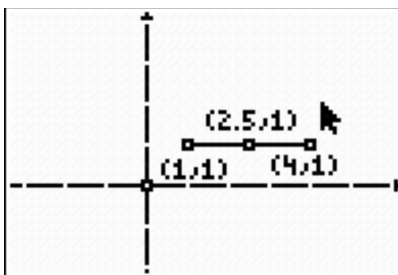
Step 2:

- Students will select **Coord. #38; Eq.** and show the coordinates for the endpoints of the segment.

If the coordinates of the endpoints are not integers, they need to use the **Hand** tool to drag the endpoints until the coordinates are integers.

Step 3:

- Students should now make a prediction about the coordinates for the midpoint of the segment.
- To check their predictions, students will select **Midpoint**, construct the midpoint of the segment, and then show its coordinates.



Step 4:

- Before moving on, students need to hide the coordinates of the midpoint with the **Hide/Show #62; Object** tool.
- They should use the **Hand** tool to drag the segment to another location. If you drag the entire segment, it will remain horizontal.

Students can make a prediction about the new coordinates of the midpoint and check their prediction by showing the coordinates of the midpoint.

Step 5:

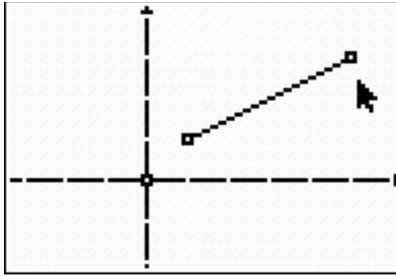
- Repeat this exploration with a new segment. Use a vertical segment.

If desired, have students explore midpoints of segments whose endpoints do not have integer coordinates, or are not in Quadrant 1.

Problem 2 – Midpoints of Diagonal Segments

Step 1:

- Instruct students to open a new Cabri Jr. file. If needed, select **Hide/Show #62; Axes** to show the coordinate axes.
- They should begin by using the **Segment** tool to construct a diagonal segment in the first quadrant.



Step 2:

- Students need to select **Coord. #38; Eq.** and show the coordinates for the endpoints of the segment.

If the coordinates of the endpoints are not integers, they should use the **Hand** tool to drag the endpoints to make the coordinates integers.

Step 3:

- Students should now make a prediction about the coordinates for the midpoint of the segment.

To check their predictions, students will construct the midpoint of the segment and show the coordinates of the midpoint.

Step 4:

- Students should now hide the coordinates of the midpoint with the **Hide/Show #62; Object** tool.
- Using the **Hand** Tool, students should drag the segment to another location. If the entire segment is selected, it will keep the same diagonal slant.

Students will make a prediction about the new coordinates of the midpoint and check their prediction by showing the coordinates of the midpoint.

Step 5:

- Repeat this exploration with a new segment.

If desired, direct students to again consider segments whose endpoints do not have integer coordinates or are not in Quadrant 1.

Step 6:

- In pairs, students should discuss the following topic:

Describe in words how to find the coordinates of the midpoint of a segment if you know the coordinates of the endpoints. Try to write a formula or a rule for midpoints.

2.2 Vertical and Adjacent Angles

This activity is intended to supplement Geometry, Chapter 1, Lesson 5.

ID: 10893

Time required: 15 minutes

Activity Overview

In this introductory or review activity, students will explore vertical and adjacent angles. They will define and identify pairs of angles. Then they will change the intersecting lines of a geometric model to make conjectures about the relationships of the pairs of angles.

Topic: Points, Lines #38; Planes

- Congruency of vertical angles
- Adjacent angles formed by two intersecting lines are supplementary.

Teacher Preparation and Notes

- This activity was written to be explored with Cabri Jr.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=10893> and select VERTICAL.8xv.

Associated Materials

- Student Worksheet: Vertical and Adjacent Angles <http://www.ck12.org/flexr/chapter/9686>, scroll down to the second activity.
- Cabri Jr. Application
- VERTICAL.8xv

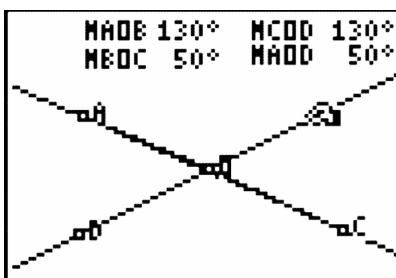
Problem 1 – Exploring Vertical Angles

Students should first define the term *vertical angles* using their textbook or other source. After opening Cabri Jr., student should press $Y =$ and select Open and then VERTICAL to view the file. Introduce the geometric model (two intersecting lines).

Students should name the two pairs of vertical angles of the model.

Direct students to explore the model independently, by grabbing and dragging points B and/or C . To grab a point, move the cursor over the point and then press a. The cursor will change to a closed fist. Press a again to release the point.

Then they need to answer the questions on the worksheet.



Problem 2 – Exploring Adjacent Angles

Students are to repeat the steps from Problem 1 with adjacent angles. They will need to first define the term *adjacent angles* using a textbook or other source.

Students are to use the same geometric model of intersecting lines from Problem 1. They need to name the four pairs of adjacent angles.

Have them explore the model independently and make a conjecture about adjacent angles.

They can also use the **Calculate** tool from the *F5* menu to add the pairs of angles confirming that adjacent angles are supplementary. To do this, select one angle measurement, press +, and then select the adjacent angle measurement.

Solutions

- Two angles whose sides are opposite rays.
- $\angle AOB$ and $\angle COD$; $\angle BOC$ and $\angle AOD$
- Sample answers.

TABLE 2.1:

Location	1 st	2 nd	3 rd	4 th
$m\angle AOB$	130.6	118.5	90.4	79.4
$m\angle BOC$	49.4	61.5	89.6	110.6
$m\angle COD$	130.6	118.5	90.4	79.4
$m\angle AOD$	49.4	61.5	89.6	100.6

- If $\angle AOD$ and $\angle BOC$ are vertical angles, then the $m\angle AOD = m\angle BOC$.
- If $\angle AOB$ and $\angle COD$ are vertical angles, then the $m\angle AOB = m\angle COD$.
- Vertical angles are congruent.
- Adjacent angles are two coplanar angles that have a common side and a common vertex but no common interior points.
- $\angle AOB$ and $\angle BOC$; $\angle BOC$ and $\angle COD$; $\angle AOD$ and $\angle COD$; $\angle AOD$ and $\angle AOB$
- Adjacent angles formed by two intersecting lines are supplementary.
- If $\angle AOB$ and $\angle BOC$ are adjacent angles formed by two intersecting lines, then $\angle AOB$ and $\angle BOC$ are supplementary.
-

2.2. VERTICAL AND ADJACENT ANGLES

$$3x = 75$$

$$x = 25$$

$$y = 180 - 75 = 105$$

12.

$$x + 10 = 4x - 35$$

$$3x = 45$$

$$x = 15$$

CHAPTER **3** **TE Reasoning and Proof - TI**

CHAPTER OUTLINE

3.1 CONDITIONAL STATEMENTS

3.1 Conditional Statements

This activity is intended to supplement Geometry, Chapter 2, Lesson 2.

ID: 8746

Time required: 40 minutes

Activity Overview

In this activity, students will write logical statements related to the given conditional statement. They will explore whether the statements are true or false and find counterexamples for false statements. These explorations will involve the slopes of parallel and perpendicular lines and lengths of collinear and noncollinear segments.

Topic: Inductive #38; Deductive Reasoning

- Write the inverse, converse, and contrapositive statements corresponding to a given conditional statement.
- Use a counterexample to prove that a statement is false.

Teacher Preparation and Notes

- This activity is designed to be used in a high school or middle school geometry classroom.
- Before beginning this activity, students should be familiar with the terms inverse, converse, and contrapositive.
- Students will discover the following concepts:
 - Parallel lines have slopes that are equal; perpendicular lines have slopes that are opposite reciprocals (the product of the slopes is -1).
 - The Segment Addition Postulate states that $AB + BC = AC$ if B is between A and C and the points are collinear. If the points are not collinear, then $AB + BC > AC$.
- If desired, teachers can explore which of the statements in the activity are also biconditional statements (definitions that are always true).
- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=8746> and select COLSEG.8xv and NOCOLSEG.8xv.

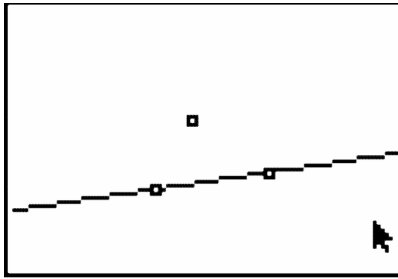
Associated Materials

- Student Worksheet: Conditional Statements <http://www.ck12.org/flexr/chapter/9687>
- Cabri Jr. Application
- COLSEG.8xv and NOCOLSEG.8xv

Problem 1 – Slopes of lines

To begin, students should open a new Cabri Jr. file.

Step 1: Students will need to construct two parallel lines. First, a **line** needs to be constructed using the Line tool and a point not on the line using the **Point** tool.



Step 2: Using the **Parallel** tool, a line parallel to the existing line should be constructed through the point not on the original line.

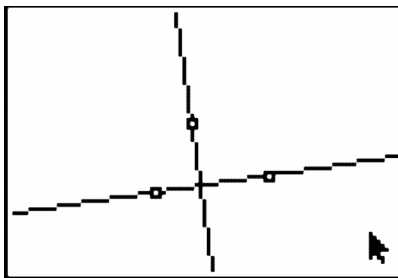
Step 3: Students will find the slope of both lines by using the **Slope** tool (**Measure #62; Slope**). Students can now use the **Hand** tool to drag the original line or the point and observe the results.

What is true of the slopes of parallel lines?

Students should record their observations on the worksheet part A . They will write the converse, inverse, and contrapositive of the statement and determine the truth of each.

Students will next construct perpendicular lines in a new *Cabri Jr.* file.

Step 4: Again construct a line and a point not on the line. Have students select the **Perp.** tool to construct a line perpendicular to the existing line through the point not on the original line.



Step 5: Students will find the slope of both lines. They should drag the original line or the point and observe the results.

Step 6: Have students select **Calculate** to find the product of the slopes. They should again drag the line to observe the results.

What is true of the slopes of perpendicular lines?

Students should record their observations, write conditional statements, and determine the truth of the statements on the worksheet in part B .

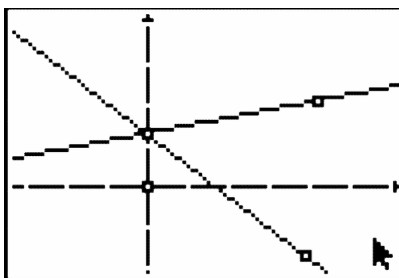
On a new Cabri Jr. file, students will construct two lines that have the same y - intercept. If the axes are not currently showing, they should select **Hide/Show #62; Axes**.

Step 7: Students should start by placing a point on the y - axis using the **Point #62; Point On** tool.

Step 8: Make sure that students construct two different lines with the same y - intercept (the point previously created on the y - axis) and find the slopes of both lines.

Note: If desired, students can display the equations of both lines using the **Coord. #38; Eq.** tool instead of the slopes.

3.1. CONDITIONAL STATEMENTS



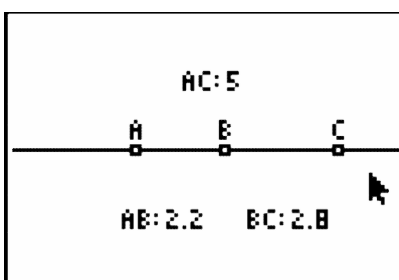
Step 9: Direct students to drag the lines and the y -intercept point and observe the changes in the coordinates and slopes.

Students will write conditional statements on the worksheet part *C*. They should sketch counterexamples for any false statements.

The screenshot at right shows a counterexample for the converse statement “If two lines have different slopes, then they have the same y -intercept.”

Problem 2 – Collinear and noncollinear segments

Step 1: Distribute the file *COLSEG* to students. Points A , B , and C are constructed on a line and the lengths AB , BC , and AC are displayed.



Step 2: Students will drag the points with the **Hand** tool and observe the changes in the lengths.

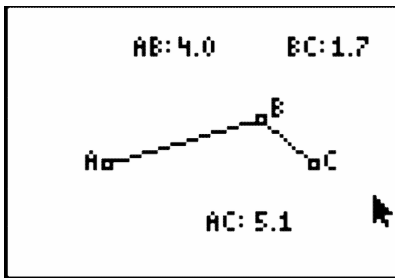
Be sure that students drag all three points, and change the order of the points (they should investigate situations where B is not between the other two points).

Students should record their observations on the worksheet and write a conditional statement.

Step 3: Distribute the file *NOCOLSEG* to students. Points A , B , and C are not collinear. The lengths AB , BC , and AC are displayed.

Step 4: Students should now drag the points and observe the changes in the distances. If desired, students can use the **Calculate** tool to find the sum of $AB + BC$.

Students should record observations on the worksheet and write a conditional statement.



Solutions

Problem 1

A. If two lines are parallel, then the slopes of the lines are equal.

Converse: If the slopes of the lines are equal, then the two lines are parallel.

Inverse: If the two lines are not parallel, then the slopes of the lines are not equal.

Contrapositive: If the slopes of the lines are not equal, then the two lines are not parallel.

Each conditional statement is true.

B. If two lines are perpendicular, then the slopes of the lines are equal to -1.

Converse: If the slopes of two lines are equal to -1, then the lines are perpendicular.

Inverse: If two lines are not perpendicular, then the slopes of the lines are not equal to -1.

Contrapositive: If the slopes of two lines are not equal to -1, then the lines are not perpendicular.

Each conditional statement is true.

C. *Converse:* If two lines have different slopes, then the lines have the same y - intercept.

Inverse: If two different lines do not have the same y - intercept, then the lines do not have different slopes.

Contrapositive: If two lines do not have different slopes, then the different lines do not have the same y - intercept.

Each conditional statement is false.

Problem 2

A. Answers will vary for distances of AB , BC , and AC .

The lengths AB and BC add up to equal AC when B is between A and C .

If A , B , and C are collinear and B is between A and C , then $AB + BC = AC$.

B. Answers will vary for distances of AB , BC , and AC .

$$AB + BC > AC$$

If A , B , and C are not collinear, then the sum of the lengths of AB and BC is greater than AC .

CHAPTER

4**TE Parallel and Perpendicular
Lines - TI****CHAPTER OUTLINE**

4.1 PARALLEL LINES CUT BY A TRANSVERSAL**4.2 TRANSVERSALS****4.3 PERPENDICULAR SLOPES**

4.1 Parallel Lines cut by a Transversal

This activity is intended to supplement Geometry, Chapter 3, Lesson 2.

ID: 8681

Time required: 30 minutes

Topic: Points, Lines #38; Planes

- *Construct two parallel lines and an intersecting transversal and measure the angles to discover that pairs of alternate angles (interior and exterior) are equal and same side angles (interior and exterior) are supplementary.*

Activity Overview

In this activity, students will develop and strengthen their knowledge about the angles formed when parallel lines are cut by a transversal. The measurement tool helps them discover the relationships between and among the various angles in the figure. Students will draw parallel lines, draw a transversal, and explore angle relationships.

Teacher Preparation

- *This activity is designed to be used in a high-school geometry classroom. It can also be used with advanced middle school students.*
- *Students should already be familiar with parallel lines as well as reading and naming angles.*
- *The screenshots on pages 1–3 demonstrate expected student results.*

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*

Associated Materials

- Student Worksheet: Parallel Lines cut by a Transversal <http://www.ck12.org/flexr/chapter/9688>
- Cabri Jr. Application

See the student worksheet on how to complete this activity.

4.2 Transversals

This activity is intended to supplement Geometry, Chapter 3, Lesson 3.

ID: 10989

Time required: 15 minutes

Activity Overview

In this activity, students will explore corresponding, alternate interior and same-side interior angles. This is an introductory activity where students will need to know how to grab points in Cabri Jr.

Topic: Points, Lines #38; Planes

- Corresponding angles are congruent
- Alternate Interior angles are congruent
- Same-Side Interior angles are supplementary

Teacher Preparation and Notes

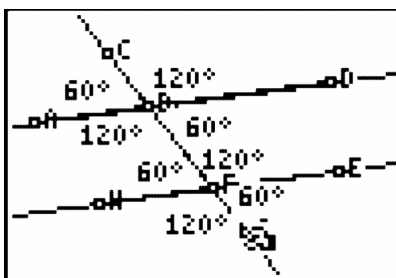
- This activity was written to be explored with Cabri Jr.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=10989> and select TRNSVRSL.8xv.

Associated Materials

- Student Worksheet: Transversals <http://www.ck12.org/flexr/chapter/9688>, scroll down to the second activity.
- Cabri Jr. Application
- TRNSVRSL.8xv

Exploring Parallel Lines cut by a Transversal

Before beginning the activity, students should know the definition of corresponding, alternate interior and same-side interior angles. Questions 1, 2, and 3 ask students to name pairs of angles from the diagram. This should be done without the use of the calculator.



Students will now run the Cabri Jr. App and open the file **TRNSVRSL**. To open a file, they should press $Y =$ and select Open.

By moving point G , students will discover the properties of two parallel lines cut by a transversal. To move a point, students need to move the cursor over the point (a square) and press **ALPHA**.

For Questions 4, 5, and 6, students will move point G to four different places. They should record the angle measurements in the tables on the worksheet. Then, students should try to generalize their results in the Conjecture section.

There are two application problems at the end of the worksheet for students to apply what they have learned in the activity. These problems can be done as homework.

Solutions

1. $\angle 4$ and $\angle 5$ is another pair
2. $\angle 4$ and $\angle 6$ is another pair
3. $\angle 4$ and $\angle 8$ is another pair
4. a. Corresponding
- b. Sample measurements.

TABLE 4.1:

	1 st position	2 nd position	3 rd position	4 th position
$m\angle ABC$	109	84	56	37
$m\angle HFB$	109	84	56	37

- c. Congruent
5. a. Same-Side Interior
- b. Sample measurements.

TABLE 4.2:

	1 st position	2 nd position	3 rd position	4 th position
$m\angle ABF$	150	136	112	75
$m\angle HFB$	30	44	68	105

- c. Supplementary
6. a. Alternate Interior
- b. Sample measurements.

TABLE 4.3:

	1 st position	2 nd position	3 rd position	4 th position
$m\angle DBF$	43	60	108	125
$m\angle BFH$	43	60	108	125

- c. Congruent

4.2. TRANSVERSALS

Conjectures

For parallel lines and a transversal...

7. if two angles are corresponding angles, then they are congruent.
8. if two angles are alternate interior angles, then they are congruent.
9. if two angles are same-side interior angles, then they are supplementary.

Extra Problems

10. $\angle 1$, $\angle 2$, and $\angle 3$ are all equal to 55°
- 11.

$$108 = 7x - 4 \quad \text{and} \quad y = 72$$

$$112 = 7x$$

$$16 = x$$

4.3 Perpendicular Slopes

This activity is intended to supplement Geometry, Chapter 3, Lesson 4.

ID: 8973

Time required: 45 minutes

Topic: Linear Functions

- *Graph lines whose slopes are negative reciprocals and measure the angles to verify they are perpendicular.*

Activity Overview

In this activity, students investigate the “negative reciprocal” relationship between the slopes of perpendicular lines. The final phase of the activity is appropriate for more advanced students as they are led through an algebraic proof of the relationship. Optional geometric activities (Problems 5 and 6) use the result to verify that (1) the radius of a circle and its tangent line are perpendicular and (2) a triangle inscribed in a circle with the diameter as one side is a right triangle.

Teacher Preparation

This activity is appropriate for students in Algebra 1. It is assumed that students have recently been introduced to the notion of slope and perhaps the fact that two lines are parallel if and only if they have the same slope.

Classroom Management

- *This activity is designed to have students explore **individually and in pairs**. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on the slope of perpendicular lines.*
- *This worksheet is intended to guide students through the main ideas of the activity. You may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=8973> and select PERP1-PERP6*

Associated Materials

- Student Worksheet: Perpendicular Slopes <http://www.ck12.org/flexr/chapter/9688>, scroll down to the third activity.
- Cabri Jr. Application
- PERP1.8xv, PERP2.8xv, PERP3.8xv, PERP4.8xv, PERP5.8xv, and PERP6.8xv

See the student worksheet on how to complete this activity.

CHAPTER

5**TE Congruent Triangles - TI****CHAPTER OUTLINE**

5.1 INTERIOR AND EXTERIOR ANGLES OF A TRIANGLE

5.2 CONGRUENT TRIANGLES

5.3 TRIANGLE SIDES AND ANGLES

5.1 Interior and Exterior Angles of a Triangle

This activity is intended to supplement Geometry, Chapter 4, Lesson 1.

ID: 8775

Time required: 40 minutes

Activity Overview

In this activity, students will measure interior and exterior angles of a triangle and make conjectures about their relationships.

Topic: Triangles #38; Congruence

- Use inductive reasoning to conjecture a theorem about the total measures of a triangle's interior angles.
- Prove that the sum of the measures of the interior angles of a triangle is 180° .
- Prove that the sum of the measures of the exterior angles of a triangle is 360° .

Teacher Preparation

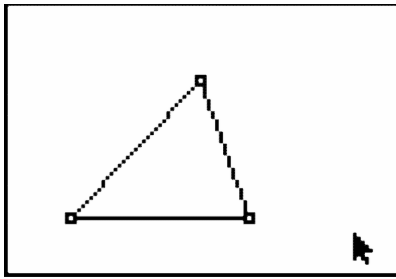
- This activity is designed to be used in a high school or middle school geometry classroom.
- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.
- If needed, review with students the definitions of the following angles before the activity: Interior angle, exterior angle, remote interior angle, adjacent interior angle.
- **Note:** Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:
 1. Move the cursor over the coordinate value so it is highlighted.
 2. Press + to display additional decimal digits or - to hide digits.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.

Associated Materials

- Student Worksheet: Interior and Exterior Angles <http://www.ck12.org/flexr/chapter/9689>.
- Cabri Jr. Application

Problem 1 – Interior angles of a triangle

Students should open a new Cabri Jr. file. They will construct a triangle using the **Triangle** tool.



Select the **Alpha-Num** tool to label the vertices A , B , and C as shown.

Note: Press **ENTER** to start the label, then press **ENTER** again to end the label.

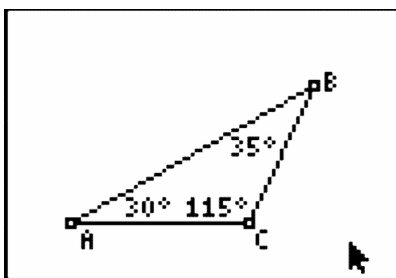
Have students measure the three interior angles of the triangle using the **Measure #62; Angle** tool.

Note: To measure an angle, press **ENTER** on three points, with the vertex of the angle being the second point selected.

Students should record this data in the first row of the chart on the student worksheet.

Students should drag a vertex of the triangle to change the angle measures. Have them try to create different types of triangles (acute, obtuse, right) and record two more sets of data in the chart.

From here, they should make a conjecture about the three interior angles.



Instruct students to use the **Calculate** tool to find the sum of the three interior angles of the triangle.

Drag a vertex and observe the results. Ask: *Do the results support your conjecture?*

Have students save the file as *IntAngle*.

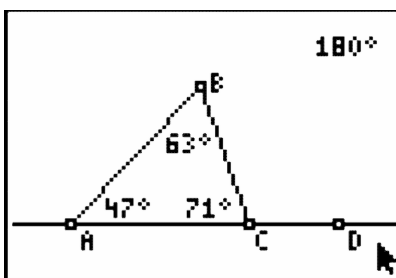
Problem 2 – One exterior angle of a triangle

Students should continue using the previous file, but instruct them to save the file as *ExtAngle* using the **Save As** option.

Have students construct a line through the two lower vertices of the triangle (A and C) using the **Line** tool.

Note: To be certain that the line passes through a vertex, be sure that the vertex point is flashing before pressing **ENTER**.

Next, students should create a new point on the line to the right of the triangle using the **Point #62; Point On** tool. Label it D .



Direct students to measure the exterior angle $\angle BCD$ with the **Measure #62; Angle** tool. Record this measure and the measures of the interior angles on the chart on the student worksheet.

They can then drag a vertex of the triangle to change the angle measures, and add two more sets of data to the chart. Students should now make some observations about the exterior angle and its relationship to other angles in the chart.

Encourage them to make calculations as needed to test their conjectures. If needed, suggest that they calculate the sums of pairs of angles in the chart.

Again, students can drag a vertex and observe the results. Ask: *Do the results support your conjectures?*

Before proceeding, have students save the file as *ExtAngle*.

Problem 3 – Three exterior angles of a triangle

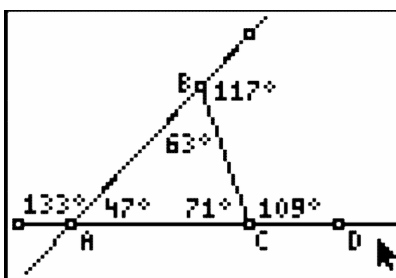
Students should continue using the previous file, but instruct them to save the file as *ExtAng3* using the **Save As** tool.

Note: Press the **ALPHA** button to access the numeric character “3.”

Students will select the **Line** tool and construct a line through vertices *A* and *B* of the triangle as shown below.

Next, students should measure one exterior angle at each vertex.

Note: It is not necessary to create an additional point on the line before measuring the angle.



Have them record the measures of the three exterior angles into the chart on the student worksheet.

They can then drag a vertex of the triangle and record more data into the chart.

Instruct students to make a conjecture about the three exterior angles, and have them calculate the sum of the three exterior angles.

Tell them to drag a vertex and observe the results. Ask: *Do the results support your conjectures?*

5.1. INTERIOR AND EXTERIOR ANGLES OF A TRIANGLE

Solutions

1. $\angle B = 43^\circ$, $\angle BCD = 143^\circ$
2. $\angle BCA = 117^\circ$, $\angle B = 32^\circ$
3. $\angle BCA = 81^\circ$, int $\angle A = 42^\circ$, ext $\angle A = 138^\circ$
4. $\angle BCA = 83^\circ$, ext $\angle A = 140^\circ$, $\angle ABC = 57^\circ$, ext $\angle B = 123^\circ$

5.2 Congruent Triangles

This activity is intended to supplement Geometry, Chapter 4, Lesson 4.

ID: 8817

Time required: 45 minutes

Topic: Triangles #38; Congruence

- Investigate the SSS, SAS, and ASA sets of conditions for congruent triangles.

Activity Overview

*In this activity, students will explore the results when a new triangle is created from an original triangle using the SSS, SAS, and ASA sets of conditions for congruence. In doing so, they will use the Cabri Jr. **Compass** tool to copy a segment and the **Rotation** tool to copy an angle.*

Teacher Preparation

- This activity is designed to be used in a high school or middle school geometry classroom.
- The **Compass** tool copies a given length as a radius of a circle to a new location.
- The screenshots on pages 1–7 demonstrate expected student results.

Classroom Management

- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress. It is recommended that you print out the following pages as a reference for students.
- If desired, have students work in groups with each group focusing on one of the problems.
- The student worksheet helps guide students through the activity and provides a place for students to record their answers and observations.
- The student worksheet includes an optional extension problem to investigate two corresponding sides and the NON-included angle.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.

Associated Materials

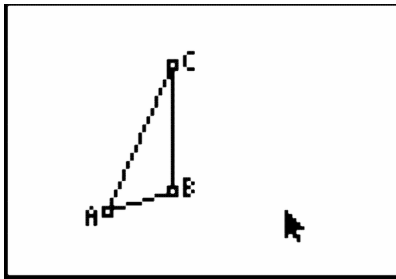
- Student Worksheet: Congruent Triangles <http://www.ck12.org/flexr/chapter/9689>, scroll down to the second activity.
- Cabri Jr. Application

Problem 1 – Three Corresponding Sides (SSS)

Step 1: Students should open a new Cabri Jr. file.

They will first construct a scalene triangle using the **Triangle** tool.

Step 2: Select the **Alph-Num** tool to label the vertices A , B , and C as shown.

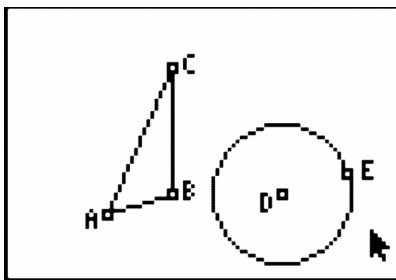


Step 3: Have students construct a free point on the screen using the **Point** tool. Label it point D with the **Alph-Num** tool.

Step 4: Students will select the **Compass** tool to copy \overline{AB} to point D .

- Press **ENTER** on \overline{AB} . A dashed circle will appear and follow the pointer.
- Press **ENTER** on point D . The compass circle is anchored at center D .

Have students construct a point on the compass circle and label it point E .



Step 5: Direct students to create \overline{DE} with the segment tool. This segment is a copy of \overline{AB} .

Hide the compass circle with the **Hide/Show #62; Object** tool.

Save this file as *CongTri*. This setup will be used again for Problems 2 and 3.

Step 6: Drag point E , and observe that the location of \overline{DE} can change but it will not change in length. You must drag either point A or point B to change the length of \overline{DE} .

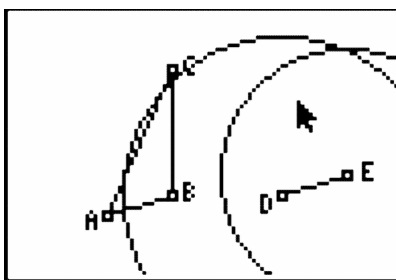
Step 7: To copy the other two segments of $\triangle ABC$, repeat the use of the **Compass** tool.

Students should copy \overline{AC} to point D .

Students should copy \overline{BC} to point E .

Find the intersection point of the two compass circles and hide the circles. Label the intersection point as F .

Finally, construct segments \overline{DF} and \overline{EF} to complete the new triangle.



Step 8: Students will now investigate whether $\triangle DEF$ is congruent to $\triangle ABC$.

Measure the sides and angles of both triangles to confirm that all corresponding parts are congruent. Use the **Measure #62; D. #38; Length** and **Measure #62; Angle** tools.

Step 9: Drag vertices of the triangles and observe the results.

Dragging vertices of $\triangle ABC$ will cause both triangles to change in size and shape.

Dragging vertices of $\triangle DEF$ will cause $\triangle DEF$ to change its location but its size and shape will remain the same as $\triangle ABC$. Notice that not all vertices of $\triangle DEF$ are available to be dragged, since they have been constructed to be dependent on $\triangle ABC$.

If students wish to save their work, use the **Save As** tool and *do not* reuse the name *CongTri*.

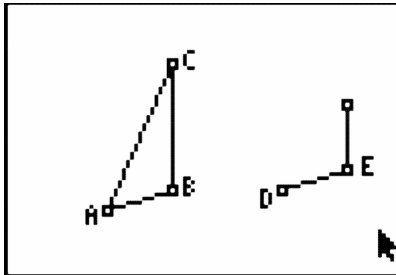
Problem 2 – Two Corresponding Sides and the Included Angle (SAS)

Step 1: Students should open the Cabri Jr. file *CongTri* that they created in Problem 1.

Recall that \overline{DE} is a copy of \overline{AB} .

Step 2: Students will select the **Rotation** tool to copy $\angle ABC$.

- Press **ENTER** on point E . This is the center of rotation.
- Press **ENTER** on \overline{DE} . This is the object to be rotated.
- Press **ENTER** three times on the vertices A , B , and C in that order to identify the angle of rotation. A new rotated segment appears.



Step 3: Have students use the **Line** tool to construct a line from point E through the endpoint of the new segment.

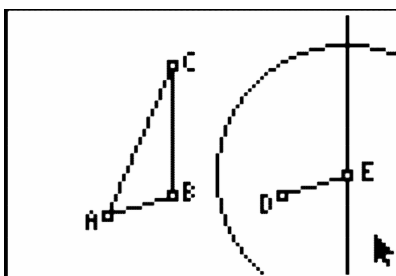
Use the **Hide/Show #62; Object** tool to hide the endpoint of the rotated segment.

Step 4: Use the **Compass** tool to copy \overline{BC} to point E .

Create the intersection point of the compass circle and the line and label it point F .

Hide the compass circle and the line.

Finally, construct \overline{DF} and \overline{EF} to complete the new triangle.



Step 5: Students will now investigate whether $\triangle DEF$ is congruent to $\triangle ABC$.

Measure the sides and angles of both triangles to confirm that all corresponding parts are congruent.

Drag vertices of the triangles and observe the results.

If students wish to save their work, use the **Save As** tool and *do not* reuse the name *CongTri*.

Problem 3 – Two Corresponding Angles and the Included Side (ASA)

Step 1: Students should open the Cabri Jr. file *CongTri* that they created in Problem 1.

Recall that \overline{DE} is a copy of \overline{AB} .

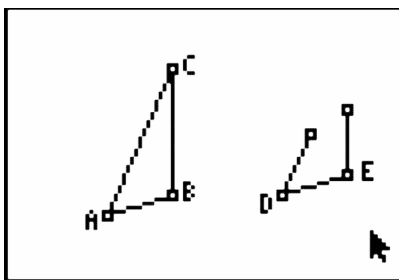
Step 2: Students will select the **Rotation** tool to copy $\angle ABC$.

- Press **ENTER** on point E . This is the center of rotation.
- Press **ENTER** on \overline{DE} . This is the object to be rotated.

Press **ENTER** three times on the vertices A , B , and C in that order to identify the angle of rotation. A new rotated segment appears.

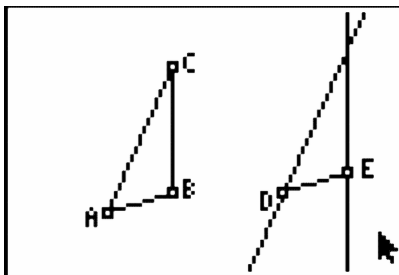
Step 3: Students will select the **Rotation** tool to copy $\angle BAC$.

- Press **ENTER** on point D . This is the center of rotation.
- Press **ENTER** on \overline{DE} . This is the object to be rotated.
- Press **ENTER** three times on the vertices B , A , and C in that order to identify the angle of rotation. A new rotated segment appears.



Step 4: Have students use the **Line** tool to construct lines over the new segments.

Use the **Hide/Show #62; Object** tool to hide the endpoints of the rotated segments.



Step 5: Create the intersection point of the two lines and label it point F . Hide the lines.

Finally, construct \overline{DF} and \overline{EF} to complete the new triangle.

Step 6: Students will now investigate whether $\triangle DEF$ is congruent to $\triangle ABC$.

Measure the sides and angles of both triangles to confirm that all corresponding parts are congruent.

Drag vertices of the triangles and observe the results.

If students wish to save their work, use the **Save As** tool and *do not* reuse the name *CongTri*.

5.3 Triangle Sides and Angles

ID: 8795

Time required: 40 minutes

Topic: Triangles #38; Congruence

- Classify triangles by angle measure.
- Prove and apply the Isosceles Triangle Theorem.
- Prove and apply the converse of the Isosceles Triangle Theorem.
- Recognize the relationship between the side lengths and angle measures of a triangle.

Activity Overview

In this activity, students will explore side and angle relationships in a triangle. First, students will discover where the longest (and shortest) side is located relative to the largest (and smallest) angle. Then, students will explore the Isosceles Triangle Theorem and its converse. Finally, students will determine the number of acute, right, or obtuse angles that can exist in any one triangle.

Teacher Preparation

- This activity is designed to be used in a high school or middle school geometry classroom.
- The screenshots on pages 1–3 demonstrate expected student results.

Classroom Management

- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.
- The student worksheet helps guide students through the activity and provides a place for students to record their answers and observations.
- **Note:** Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:

1. Move the cursor over the coordinate value so it is highlighted.
2. Press + to display additional decimal digits or - to hide digits.

- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.

Associated Materials

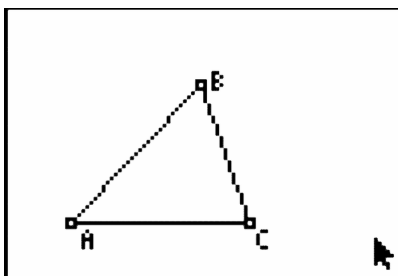
- Student Worksheet: Triangle Sides and Angles <http://www.ck12.org/flexr/chapter/9689>, scroll down to the third activity.
- Cabri Jr. Application

Problem 1 – Size and Location of Sides and Angles

Step 1: Opening a new Cabri Jr. file, students should first construct a triangle using the **Triangle** tool.

Step 2: Select the **Alph-Num** tool to label the vertices A , B , and C as shown.

Note: Press **ENTER** to start entering the label, and then press **ENTER** again to end the label.



Step 3: Students should measure the three interior angles of the triangle using the **Measure #62; Angle** tool. To measure an angle, press **ENTER** on each of three points that define the angle, with the vertex of the angle as the second point chosen.

Step 4: Direct students to measure the three side lengths using the **Measure #62; D. #38; Length** tool.

Note: To measure a side of the triangle, you must click on the endpoints of the segment. If you click on the side itself, the tool will return the *perimeter* of the triangle.

Step 5: Now students should drag a vertex of the triangle to change the angle measures and side lengths.

Encourage them to make a conjecture about the sizes and locations of the angles and sides in a triangle.

Students can answer the question on the worksheet based on their observations, concluding that the largest angle is opposite the longest side, and the smallest angle is opposite the shortest side.

Step 6: Have students save this file as *Triangle*.

Problem 2 –The Isosceles Triangle Theorem

Step 1: Distribute the Cabri Jr. files *Isostri1* and *Isostri2* to student calculators

Have students open the file *Isostri1*. An isosceles triangle has been constructed and the congruent sides have side lengths displayed.

Step 2: Students should measure all three angles using the **Measure #62; Angle** tool.

Step 3: Direct students to drag a vertex of the triangle to explore what happens to the angle measures when two sides have equal lengths.

Students should conclude that a triangle with two congruent sides also has two congruent angles. This is known as the Isosceles Triangle Theorem.

Step 4: The converse of the Isosceles Triangle Theorem may be explored in the file *Isostri2*.

Step 5: Students should measure all three sides using the **Measure #62; Length** tool, then drag a vertex to explore.

Step 6: On their worksheets, students should make the conclusion that a triangle with two congruent angles also has two congruent sides.

Problem 3 – Types of Angles in a Triangle

Step 1: Tell students to open the file *Triangle* that they saved from Problem 1.

If desired, have them hide the side lengths using the **Hide/Show #62; Object** tool.

Step 2: Students should drag a vertex and notice how many angles of each type can exist in a triangle. They should conclude that a triangle:

- can have three acute angles
- cannot have three right angles
- cannot have three obtuse angles

Challenge them to observe that a triangle can have, at most, one right or one obtuse angle and that a triangle cannot have both a right and obtuse angle.

CHAPTER 6

TE Relationships within Triangles - TI

CHAPTER OUTLINE

- 6.1 PERPENDICULAR BISECTOR**
 - 6.2 HANGING WITH THE INCENTER**
 - 6.3 BALANCING POINT**
 - 6.4 HEY ORTHO! WHAT'S YOUR ALTITUDE?**
-

6.1 Perpendicular Bisector

This activity is intended to supplement Geometry, Chapter 5, Lesson 2.

ID: 11198

Time required: 15 minutes

Activity Overview

In this activity, students will explore the perpendicular bisector theorem and discover that if a point is on the perpendicular bisector of a segment, then the point is equidistant from the endpoints. This is an introductory activity where students will need to know how to grab and move points, measure lengths, and construct the perpendicular bisector with Cabri Jr.

Topic: Triangles #38; Their Centers

- *Perpendicular Bisector Theorem*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1119> and select PERPBIS and POINTS.*

Associated Materials

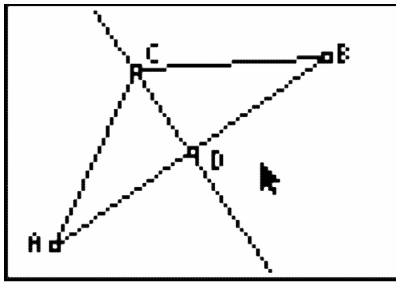
- Student Worksheet: Perpendicular Bisectors <http://www.ck12.org/flexr/chapter/9690>
- Cabri Jr. Application
- PERPBIS.8xv and POINTS.8xv

In this activity students will explore the properties of perpendicular bisectors. In the first part, students will measure the distance to a point on the bisector to identify congruent segments. In the second part, students will apply what they have learned about perpendicular bisectors to the context of map coordinates.

Problem 1 – Exploring the Perpendicular Bisector Theorem

Students will be exploring the distance from a point on the perpendicular bisector to the endpoints of a segment and discover that if a point is on the perpendicular bisector of a segment, then it is equidistant from the two endpoints of the segment.

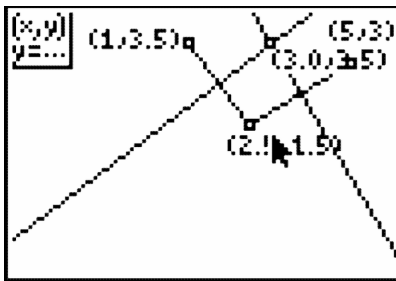
Students will measure the lengths of segments using the **Distance #38; Length** tool (press **GRAPH** and select **Measure #62; D.#38;Length**). To drag a point, students will move the cursor over the point, press **ALPHA**, move the point to the desired location, and then press **ALPHA** again to release the point.



Problem 2 – An Application of the Perpendicular Bisector Theorem

Students will apply what they learned from problem one and find the intersection of two perpendicular bisectors to find a point equidistant from three points.

To draw a segment, students should use the **Segment** tool (press **WINDOW** and select **Segment**). To draw a perpendicular bisector, press **ZOOM** and select **Perp. Bis.**.



Solutions

Problem 1 – Exploring the Perpendicular Bisector Theorem

- Answers will vary. The measures of \overline{AC} and \overline{BC} should be equal.
- Sample answer: AC is congruent to BC . The distances are the same.
- If a point is on the perpendicular bisector of a segment, then it is equidistant from the two endpoints of the segment.

Problem 2 – An Application of the Perpendicular Bisector Theorem

- (6, 7)
- F7

6.2 Hanging with the Incenter

This activity is intended to supplement Geometry, Chapter 5, Lesson 3.

ID: 11360

Time Required: 45 minutes

Activity Overview

In this activity, students will explore the angle bisector theorem and discover that if a point is on the angle bisector of a segment, then the point is equidistant from the sides. This is an introductory activity, but students will need to know how to grab and move points, measure lengths, and construct the perpendicular bisector with Cabri Jr.

Topic: Triangles #38; Their Centers

- *Angle Bisector Theorem*
- *Incenter*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=11360> and select ANGBIS.*

Associated Materials

- Student Worksheet: Hanging with the Incenter <http://www.ck12.org/flexr/chapter/9690>, scroll down to the second activity.
- Cabri Jr. Application
- ANGBIS.8xv

Problem 1 – Exploring the Angle Bisector Theorem

Students will be exploring the distance from a point on the angle bisector to the segment. They will discover that if a point is on the angle bisector of an angle, then it is equidistant from the two segments of the angle.

Students will measure the lengths of segments using the **Distance #38; Length** tool (press **GRAPH** and select **Measure #62; D.#38; Length**). To drag a point, students will move the cursor over the point, press **ALPHA**, move the point to the desired location, and then press **ALPHA** again to release the point.

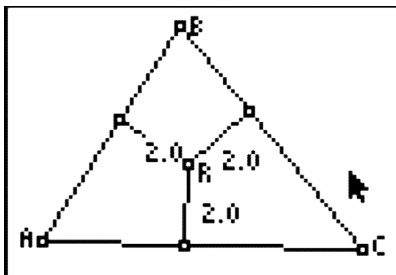
Problem 2 – Exploring the Incenter of a Triangle

Students will need to open a new *Cabri Jr.* file by pressing **Y =**, selecting **New**, and answer **no** if asked to save.

Students are to create an acute triangle and find the angle bisector of all three angles of the triangle. Students should realize that they are concurrent and answer Questions 4–8 on the accompanying worksheet.

Students will need to find the distance from the incenter to the 3 sides of the triangle. Students may need to be reminded how to find the distance from a point to a line or segment. It is the perpendicular distance from the incenter to the side. Teachers should encourage the students to hide their angle bisectors. Be sure that all students understand that the perpendicular line created from the incenter to a side of the triangle is not the perpendicular bisector.

Students should discover that all of these distances are equal.



Problem 3 – Extension

Problem 3 is an extension of this activity. Students will use the handheld and the angle bisectors to find the coordinates of fence posts on a plot of land. Note: This problem will be very hard for the average student.

Students must first set up coordinates for the 2 fences at $(-1, 2)$, $(1, 0)$, $(1, 3)$, and $(5, 3)$. Next, students should create segments for the two fences. From here, there are several approaches to solving this problem.

Using the incenter:

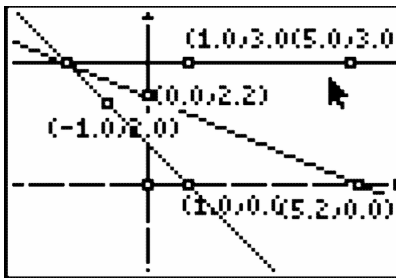
One approach is to find the incenter of the triangle formed by the intersection of the two fences (over the pond) and a segment joining the two fences. Students should construct a segment from $(1, 0)$ to $(5, 3)$ connecting the two fences. This is to create two angles that they can use to find angle bisectors. The point of intersection of the two angle bisectors is the incenter of the triangle. This point is therefore equidistant from the two fences.

However, we need two points to determine the line where the fence will be. Students will need to create a similar triangle that connects the two fences. To guarantee similarity, choose a segment that is parallel to the segment joining $(1, 0)$ and $(5, 3)$ using the **Parallel** tool (**Press ZOOM**, then scroll down to **Parallel**). Find the incenter of the second triangle and you can create the line through those two incenters. One point on the line should be at $(5.24, 0)$ or $(5.2, 0)$ and the other can be any other point on the line connecting the two incenters. The y -intercept is another convenient point to find the coordinates of $(0, 2.2)$.

Using only an angle bisector:

Another approach is to find the point of intersection of the fences (if they were continued over the pond). Students can construct lines for the Fence 1 and Fence 2 and plot the intersection point of the lines. Then, students can use the **Angle Bisector** tool to find the angle bisector of this angle. This ray will be equidistant from Fence 1 and Fence 2. Students can plot a point on this ray and display and record the coordinates of this point. Then, move the point and record the new coordinates.

6.2. HANGING WITH THE INCENTER



Solutions

1. Sample answers

TABLE 6.1:

Position	1st position	2nd position	3rd position	4th position
<i>DF</i>	2.25473	2.72284	1.85757	1.12521
<i>DE</i>	2.25473	2.72284	1.85757	1.12521

2. They are equal.
3. Equidistant
4. They are concurrent.
5. Not possible
6. Not possible
7. Obtuse, Acute, and Right
8. They are all equal.
9. Sample answer: (5.2, 0) and (0, 2.2)

6.3 Balancing Point

This activity is intended to supplement Geometry, Chapter 5, Lesson 4.

ID: 11403

Time Required: 45 minutes

Activity Overview

In this activity, students will explore the median and the centroid of a triangle. Students will discover that the medians of a triangle are concurrent. The point of concurrency is the centroid. Students should discover that a triangle's center of mass and centroid are the same.

Topic: Triangles #38; Their Centers

- *medians*
- *centroid*

Teacher Preparation and Notes

- *This activity was written to be explored with the TI-84 using the Cabri Jr. application.*
- *This is an introductory activity for which students will need to know how to construct triangles, grab and move points, measure lengths, and construct segments.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1140> and select **CENTROID, MEDIAL, and MIDSEG**.*

Associated Materials

- Student Worksheet: Balancing Point <http://www.ck12.org/flexr/chapter/9690>, scroll down to the third activity.
- Cabri Jr. Application
- CENTROID.8xv, MEDIAL.8xv, MIDSEG.8xv

Problem 1 – Exploring the Centroid of a Triangle

Students are to cut out the triangle on their worksheet and attempt to balance it with their pencil. Students will mark the balancing point on their triangle.

Students will then use their handhelds to find the point where the triangle will be balanced. They will be asked to compare their findings to see if their point was close to the point given by the calculator. Explain to the students that the balancing point for an object is called the center of mass.

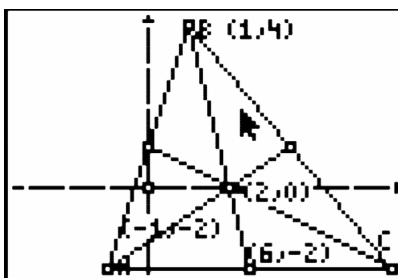
Problem 2 – Exploring the Medians of a Triangle

Students will need to define the terms *median of a triangle* and *centroid* from either their textbook or another source.

Students will then create the three medians of the triangle given in *Centroid.8xv*. Students will need to find the midpoint of the three sides of a triangle. Students can access the **Midpoint** tool by selecting **ZOOM #62; Midpoint**. Then, students can click on a side of the triangle to place the midpoint.

Next, students should construct the segment connecting point A and the opposite midpoint. Students can find the **Segment** tool by selecting **WINDOW #62; Segment**. They should repeat this for the other two medians.

Students are asked several questions on their accompanying worksheet. Students should be able to find the centroid of a triangle and understand that this point is the center of mass.



Problem 3 – Extending the Centroid

Students will extend the concept of the centroid in Problem 3. First, define the **medial triangle** to be the triangle formed by connecting the midpoints of the sides of a triangle.

In *Medial.8xv*, students will see a triangle and its centroid. Student should construct the medial triangle and the centroid of the medial triangle.

Students will notice that the centroid of the original triangle and the centroid of the medial triangle are the same.

Problem 4 – Extending the Median

In Problem 4, students will extend the concept of the median. Tell students that the midsegment is a line segment joining the midpoints of two sides of a triangle.

In *Midseg.8xv*, students will see $\triangle ABC$ with midpoints D , E , and F of sides \overline{AC} , \overline{AB} , and \overline{BC} , respectively.

Students should then construct the median that extends from point A to point F . Next, they should construct the intersection point, G , of the median and the midsegment.

Students should find the lengths of \overline{AG} , \overline{FG} , \overline{DG} , and \overline{EG} . Students can find the **Length** tool by selecting **GRAPH #62; Measure #62; D. #38; Length**.

Students will notice that the median and midsegment bisect each other.

Solutions

1. Student solutions will vary. Students' balancing points should be close to $(2, 0)$.
2. The median of a triangle is the segment joining the vertices of a triangle to the midpoint of the opposite side.
3. They are concurrent.
4. $(2, 0)$

5. They are close.
6. $(\frac{-1+1+6}{3}, \frac{-2+4-2}{3}) = (2, 0)$
7. Centroid is the point of concurrency of the medians.
8. They are the same.
9. They bisect each other.

6.4 Hey Ortho! What's your Altitude?

This activity is intended to supplement Geometry, Chapter 5, Lesson 4.

ID: 11483

Time Required: 45 minutes

Activity Overview

In this activity, students will explore the altitudes of a triangle. Students will discover that the altitude can be inside, outside, or a side of the triangle. Students will discover that the altitudes are concurrent. The point of concurrency is the orthocenter. Students should discover the relationship between the type of triangle and the location of the point of concurrency. Students will discover properties of the orthocenter in equilateral triangles.

Topic: Triangles #38; Their Centers

- *Altitudes of a Triangle*
- *Orthocenter*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1148> and select ACUTE, EQUILATE, MEDIAL2, OBTUSE and TRIANGLE.*

Associated Materials

- Student Worksheet: Hey Ortho! Whats your Altitude? <http://www.ck12.org/flexr/chapter/9690>, scroll down to the fourth activity.
- Cabri Jr. Application
- ACUTE.8xv, OBTUSE.8xv, TRIANGLE.8xv, EQUILATE.8xv, and MEDIAL2.8xv

Problem 1 – Exploring the Altitude of a Triangle

Students should define the altitude of a triangle using their textbooks or other source on their worksheet.

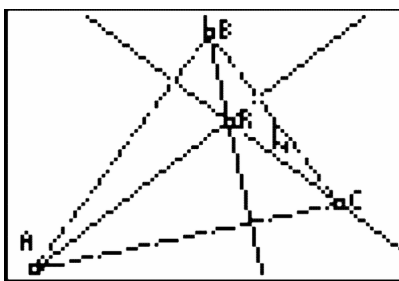
In files *ACUTE* and *OBTUSE*, students are to create the altitude of an acute triangle, obtuse triangle, and right triangle, respectively. Students will need to construct the perpendicular line to a line through a point (press **ZOOM** and select **Perp.**).

To show the altitude of the triangle, students will need to find the intersection point of the perpendicular line and the line that extends from the base of the triangle. Students will then need to create a segment from the opposite vertex to the intersection point.

Problem 2 – Exploring the Orthocenter

In file *TRIANGLE*, students are given $\triangle ABC$. They should construct the altitude of each vertex of the triangle. Students should realize that they are concurrent. Explain to students that the point of concurrency is called the orthocenter of the triangle.

Students should discover a few facts about the orthocenter. If a triangle is an acute triangle, then the orthocenter is inside of the triangle. If a triangle is a right triangle, then the orthocenter is on a side of the triangle. If a triangle is an obtuse triangle, then the orthocenter is outside of the triangle.



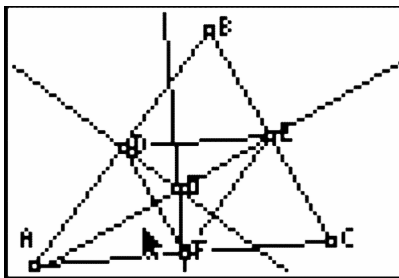
Problem 3 – Exploring the Altitude of an Equilateral Triangle

Students will need to find the distance from point P to the 3 sides of the triangle and the altitude BD in the file *EQUILATE*. The **Distance #38; Length** tool is found in the **GRAPH/F5** menu (press **GRAPH** and select **Measure #62; D.#38;Length**). Once students make their calculation, they can move it to the right of the appropriate label (EP , for instance). Students will then need to calculate the sum of $EP + FP + GP$ using the **Calculate** tool (press **GRAPH** and select **Calculate**). Students will need to move their cursor to EP until it is underlined, then press **ENTER**, then press **+**, then move their cursor to FP until it is underlined, then press **ENTER**. A measurement that you can place anywhere on the screen is given. Place the measurement by pressing **ENTER**. Repeat this process for $EP + FP$ that was just created and GP to get the sum $EP + FP + GP$. Students should discover that the sum of the distances from all three sides to point P and the length of the altitude BD are equal. Make sure that students see that this only works for an equilateral triangle.

Problem 4 – Exploring the Orthocenter of a Medial Triangle

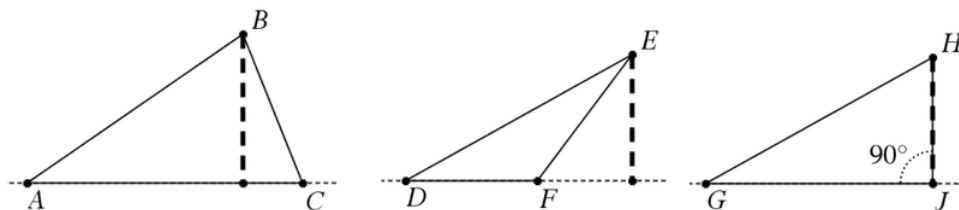
In the file *MEDIAL2*, students are given a triangle, its medial triangle, and the orthocenter of the medial triangle. Students are to discover which point of the original $\triangle ABC$ is the orthocenter of the medial triangle. Choices should include the centroid, circumcenter, incenter, and orthocenter.

Students should discover that the orthocenter of the medial triangle is the circumcenter of the original triangle.



Solutions

1. An altitude of a triangle is a perpendicular segment from a vertex to the line containing the side opposite that vertex.
- 2.



3. a. inside
- b. outside
- c. a side of
4. They are concurrent
5. Right Triangle
6. Acute Triangle
7. Obtuse Triangle
- 8.

TABLE 6.2:

Position	1 st position	2 nd position	3 rd position	4 th position
BD	4.9	5.4	5.4	5.4
$EP + FP + GP$	4.9	5.4	5.4	5.4

9. They are equal
10. equal to the length of the altitude
11. circumcenter

CHAPTER 7**TE Quadrilaterals - TI****CHAPTER OUTLINE**

7.1 PROPERTIES OF PARALLELOGRAMS

7.2 PROPERTIES OF TRAPEZOIDS AND ISOSCELES TRAPEZOIDS

7.3 PROPERTIES OF RHOMBI, KITES AND TRAPEZOIDS

7.1 Properties of Parallelograms

This activity is intended to supplement Geometry, Chapter 6, Lesson 2.

ID: 11932

Time Required: 15 minutes

Activity Overview

Students will explore the various properties of parallelograms. As an extension, students can also explore necessary and sufficient conditions that guarantee that a quadrilateral is a parallelogram.

Topic: Quadrilaterals #38; General Polygons

- *Inductive Reasoning*
- *Parallelograms*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. app on the TI-84.*
- *Before beginning this activity, make sure that all students have the Cabri Jr. application, and the Cabri Jr. files PAR1.8xv, PAR2.8xv, PAR3.8xv loaded on their TI-84 calculators.*
- *In Cabri Jr., to grab a point hover the cursor over the point and press **ALPHA**. To release press **ALPHA** or **ENTER**. To move a point after grabbing it, use the arrow keys.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1193> and select PAR, PAR2, and PAR3.*

Associated Materials

- Student Worksheet: Properties of Parallelograms <http://www.ck12.org/flexr/chapter/9691>
- Cabri Jr. Application
- PAR.8xv, PAR2.8xv, and PAR3.8xv

Problem 1 – Properties of Parallelograms

Students will begin this activity by looking at properties of parallelograms. They will discover that opposite sides are congruent, opposite angles are congruent, and that consecutive angles are supplementary.

As an extension, students can prove each of these using parallel lines and transversals. Students will need to know the properties of alternate interior angles, same-side interior angles, and corresponding angles.

Problem 2 – Diagonals of Parallelograms

In Problem 2, students are asked to investigate the diagonals of a parallelogram. Students should discover that the diagonals of a parallelogram bisect each other. This particular wording may be hard for students discover independently.

Problem 3 – Extension: Proving Parallelograms

In this problem, students can explore various properties and see if they guarantee that a quadrilateral is a parallelogram.

Students should know that the following prove that a quadrilateral is a parallelogram:

1. both pairs of opposite sides are congruent
2. both pairs of opposite angles are congruent
3. both pairs of opposite sides are parallel
4. one pair of opposite sides is both parallel and congruent
5. the diagonals bisect each other

Problem 4 – Extension: Extending the Properties

For this problem, students will create any quadrilateral and name it **GEAR**. Next, students will find the midpoint of each side and connect the midpoints to form a quadrilateral. Students will use the properties of a parallelogram to see that a parallelogram is always created.

Solutions

1. A quadrilateral with both pairs of opposite sides parallel.
2. Sample answers:

TABLE 7.1:

Position	\overline{QU}	\overline{UA}	\overline{AD}	\overline{DQ}
1	4.50	2.44	4.50	2.44
2	4.50	2.94	4.50	2.94
3	4.90	2.94	4.90	2.94
4	5.50	2.94	5.50	2.94

3. The opposite sides are congruent.
4. Sample answers:

TABLE 7.2:

Position	$\angle Q$	$\angle U$	$\angle A$	$\angle D$
1	100	80	100	80
2	106	74	106	74
3	124	56	124	56
4	77	103	77	103

5. The consecutive angles are supplementary.

6. The opposite angles are congruent.

7. Sample answers:

TABLE 7.3:

Position	\overline{QR}	\overline{AR}	\overline{DR}	\overline{RU}
1	2.84	2.84	3.29	3.29
2	2.94	2.94	3.45	3.45
3	3.06	3.06	3.58	3.58
4	3.19	3.19	3.71	3.71

8. The diagonals bisect each other.

9. Yes

10. No

11. Yes

12. Parallelogram

13. Sample: Both pairs of opposite sides are parallel.

7.2 Properties of Trapezoids and Isosceles Trapezoids

This activity is intended to supplement Geometry, Chapter 6, Lesson 5.

ID: 9084

Time required: 25 minutes

Topic: Quadrilaterals #38; General Polygons

- *Prove and apply theorems about the properties of rhombuses, kites and trapezoids.*

Activity Overview

A trapezoid is a quadrilateral where one pair of sides is parallel while the other two sides are not. In an isosceles trapezoid the non-parallel sides are congruent. In this activity we will attempt to create an isosceles trapezoid from an ordinary trapezoid, then approach the problem in a different manner and finally, examine the properties of trapezoids.

Teacher Preparation

This activity is designed to be used in a middle-school or high-school geometry classroom.

- *For this activity, students should know the definitions of a trapezoid and isosceles trapezoid. If they do not already know these terms, you can define them as they appear in the lesson, but allow extra time to do so.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*

Associated Materials

- Student Worksheet: Properties of Trapezoids and Isosceles Trapezoids <http://www.ck12.org/flexr/chapter/9691>, scroll down to the second activity.
- Cabri Jr. Application

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their graphing calculators.*
- *See the student worksheet on how to complete this activity.*

7.3 Properties of Rhombi, Kites and Trapezoids

ID: 12093

Time Required: 45 minutes

Activity Overview

In this activity, students will discover properties of the diagonals of rhombi and kites and properties of angles in rhombi, kites, and trapezoids.

Topic: Quadrilaterals #38; General Polygons

- Rhombi
- Kites
- Trapezoids

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. app on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1209> and select READ1, READ2, READ3, KING1, KING2, KING3, and TRAP.*

Associated Materials

- Student Worksheet: Properties of Rhombi, Kites and Trapezoids <http://www.ck12.org/flexr/chapter/9691>, scroll down to the third activity.
- Cabri Jr. Application
- READ1.8xv, READ2.8xv, READ3.8xv
- KING1.8xv, KING2.8xv, KING3.8xv
- TRAP.8xv

Problem 1 – Properties of Rhombi

Students will begin this activity by looking at angle properties of rhombi. They are asked several questions about the angles and diagonals of a rhombus.

An extension of this exercise would be to prove each of these using parallel lines and transversals. Students will need to know the properties of alternate interior angles, same-side interior angles, and corresponding angles.

Problem 2 – Properties of Kites

In Problem 2, students will be asked to begin looking at angle properties of kites. They are asked several questions about the angles and diagonals of kites.

Problem 3 – Properties of Trapezoids

In this problem, they will explore angle properties of trapezoids. Students are given trapezoid $TRAP$ and the measure of angles T , R , A , and P . Students will move point R to four different positions and collect the measures of angles T , R , A , and P onto their accompanying worksheet.

An extension of this exercise would be to prove leg angles are supplementary using parallel lines and transversals. Students will need to know the properties of alternate interior angles, same-side interior angles, and corresponding angles.

Solutions

1. Sample answers:

TABLE 7.4:

Position	$\angle R$	$\angle E$	$\angle A$	$\angle D$
1	76	104	76	104
2	51	129	51	128
3	95	85	95	85
4	64	116	64	116

2. supplementary

3. congruent

4. right angles

5. bisect

6. Sample answers:

TABLE 7.5:

Position	$\angle K$	$\angle I$	$\angle N$	$\angle G$
1	69	130	32	130
2	71	116	58	116
3	61	123	53	123
4	37	142	38	142

7. One pair of opposite angles is congruent and one pair of opposite angles is not congruent.

8. right angles

9. One pair of opposite angles is bisected and one set of opposite angle is not. The pair that is not congruent is the

one getting bisected.

10. Sample answers:

TABLE 7.6:

Position	<i>T</i>	<i>R</i>	<i>A</i>	<i>P</i>
1	108	72	54	126
2	126	54	45	135
3	39	141	24	156
4	57	123	37	143

11. Leg angles are supplementary.

CHAPTER 8**TE Similarity - TI****CHAPTER OUTLINE**

8.1 CONSTRUCTING SIMILAR TRIANGLES**8.2 SIDE-SPLITTER THEOREM****8.3 PERSPECTIVE DRAWING**

8.1 Constructing Similar Triangles

This activity is intended to supplement Geometry, Chapter 7, Lesson 4.

ID: 8179

Time required: 30 minutes

Activity Overview

*In this activity, students will investigate three different methods of constructing similar triangles. They will use the **Dilation** tool with the dilation point inside and outside of the triangle to investigate different relationships. Also, students will use the **Parallel** tool to construct two similar triangles from one triangle.*

Topic: Ratio, Proportion #38; Similarity

- Use inductive reasoning to classify each set of conditions as necessary and/or sufficient for similarity:

a) the side lengths of one triangle are equal to the corresponding side lengths of another triangle.

b) two triangles of one triangle are congruent to two angles of another triangle.

Teacher Preparation

- This activity is designed to be used in a high school geometry classroom.
- Before starting this activity, students should be familiar with the term dilation.
- This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.

Associated Materials

- Student Worksheet: Constructing Similar Triangles <http://www.ck12.org/flexr/chapter/9692>
- Cabri Jr. Application

Students need to press **APPS** and select **CabriJr** to start the application. When they open a new document, they need to make sure that the axes are hidden. If the axes are displayed, press **GRAPH #62; Hide/Show #62; Axes**.

Explain to students that similar triangles are those that have the same shape but not necessarily the same size.

Congruent triangles are a special type of similar triangle where corresponding sides are congruent. In similar triangles, corresponding angles are congruent but corresponding sides are proportional. In this activity, students will look at three methods of constructing similar triangles and will test these properties using dilations or stretches.

In order to examine all of the sides and angles, students should work in groups of three. Have one student (Student A on the worksheet) in each group construct the first triangle and save it as "**SIMTRI**." This is saved in the TI-84 Plus family as an **APPVAR**. Student A needs to transfer the **APPVAR** to the other two students in the group.

Problem 1 – Similar Triangles using Dilation

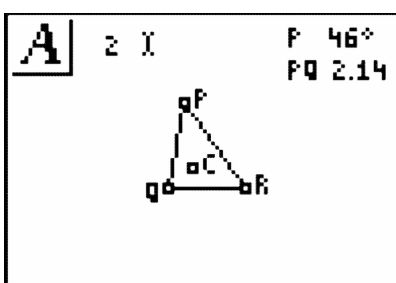
Student A will use the **Triangle** tool to construct a triangle of any shape or size. Then they need to use the **Alph-Num** tool to label the vertices P , Q , and R .

Student A should measure angle P and side PQ , Student B should measure angle Q and side QR , Student C should measure angle R and side PR .

Note: To increase the number of digits for the length of the side, hover the cursor over the measurement and press +.

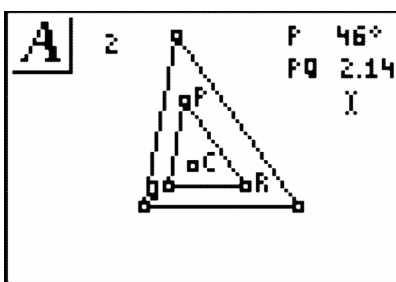
With the **Point** tool, students on their own calculators will construct a point C in the center of the triangle. They will then use the **Alph-Num** tool to place the number 2 at the top of the screen.

Explain to students that the point C will be the center of the dilation and the number 2 will be the scale factor.



With the **Dilation** tool selected, students need to move the cursor to point C , the center of the dilation, and press **ENTER**. They should notice that the shape of the cursor changes, which is to indicate that everything will be stretched away from this point. Then students move the cursor to the perimeter of the triangle and press **ENTER**. Finally, move to the scale factor, 2, and press **ENTER**.

Students will see that a new, larger triangle will appear outside of $\triangle PQR$. Have a preliminary discussion with students about whether they think the new triangle is similar to $\triangle PQR$ and how they can confirm their hypothesis.



Instruct students to label this triangle as XYZ so that X corresponds to P , Y to Q and Z to R . Each person in the group should select and measure their appropriate angle and side in the new triangle. Students should answer Questions 1 and 2 on the worksheet comparing the two angles and two sides.

Explain to students that $XY = 2 PQ$ indicates that the sides have the ratio 2:1. If all three sides display the same result, then the sides are said to be proportional.

To answer Questions 3 and 4 on the worksheet, students will observe the changes in the triangles as they drag a point in the original triangle and the point of dilation.

8.1. CONSTRUCTING SIMILAR TRIANGLES

Problem 2 – Different Scale Factors

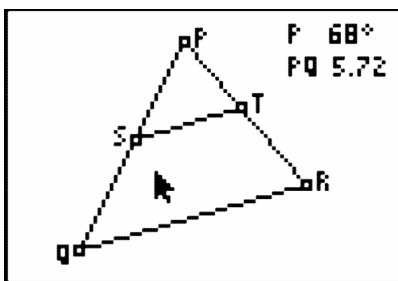
Students will continue using the same file. To change the scale factor, students must select the **Alph-Num** tool, move the cursor to the scale factor of 2 and press **ENTER**. Then they should delete 2 by pressing **DEL**, press **ALPHA** to change the character to a number, and enter 3. Students can now answer Question 5 on the worksheet.

Now students will investigate another scale factor by changing it from 3 to 0.5. They should then answer Questions 6 and 7 on the worksheet, summarizing their observations of the triangles and their measurements.

For the last investigation in this problem, students will look at the effect of a dilation when the center point C is outside of the pre-image triangle and a negative scale factor is used. Students will need to move $\triangle PQR$ by moving the cursor to one side and when all sides of the triangle flash, press **ALPHA** and then use the arrow keys.

Problem 3 – Similar Triangles with a Parallel Line

Finally, students will look at a completely different method of constructing similar triangles. All students will need to open a new file. It is the teacher's decision to have them save the file. Student A should construct a triangle PQR and transfer it to the others in their group. They will all measure their same side and angle as before.



All students need to construct a point on \overline{PQ} using the **Point on** tool and label it S using the **Alph-Num** tool. Using this point, they construct a line that is parallel to \overline{QR} by selecting the **Parallel** tool, and then choosing the point and side.

With the **Intersection** tool, students need to find the intersection of side PR and the parallel line, and then use the **Alph-Num** tool to label it as T . The **Hide/Show #62; Object** tool enables students to hide the parallel line, and then construct segment ST with the **Segment** tool. If students can prove that all of the angles are congruent, they have completed their first step in proving that the two triangles are similar.

With the **Measure #62; D. #38; Length** tool, students need to measure segments of the corresponding sides. In the screen to the right all three pairs of sides have been measured.

To complete the proof, students will need to confirm that all of the sides are proportional. Using the Calculate tool, they can select a side measurement, press the operation key (\div) and then select the corresponding side measurement.

If all three ratios are equivalent, then the sides are proportional and the two triangles are similar.

8.2 Side-Splitter Theorem

This activity is intended to supplement Geometry, Chapter 7, Lesson 5.

ID: 12318

Time Required: 15 minutes

Activity Overview

In this activity, students will explore the side-splitter theorem.

Topic: Ratio, Proportion #38; Similarity

- *Side-Splitter Theorem*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. app on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=12318> and select SIDESP1, SIDESP2, and SIDESP3.*

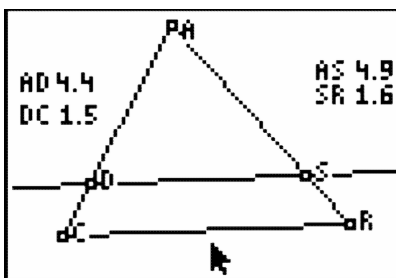
Associated Materials

- Student Worksheet: Side-Splitter Theorem <http://www.ck12.org/flexr/chapter/9692>, scroll down to the second activity.
- Cabri Jr. Application
- SIDESP1.8xv, SIDESP2.8xv, and SIDESP3.8xv

Problem 1 – Side-Splitter Theorem

Students will begin this activity by looking at the side-splitter theorem. Students are given a triangle with a segment parallel to one side. They will discover that if a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

Students will be asked to collect data by moving point A and point D . Students are asked several questions about the relationships in the triangle.



Problem 2 – Application of the Side-Splitter Theorem

In Problem 2, students will be asked to apply the side-splitter theorem to several homework problems.

Problem 3 – Extension of the Side-Splitter Theorem

In Problem 3, students will discover the corollary to the Side-Splitter Theorem: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

Students are asked several questions about the corollary to the side-splitter theorem.

Solutions

1. Sample answers:

TABLE 8.1:

Position	AD	DC	AS	SR	$\frac{AD}{DC}$	$\frac{AS}{SR}$
1	4.1	1.8	4.6	2.0	2.27	2.30
2	3	2.8	3.4	3.2	1.07	1.06
3	2.6	2.4	3.2	3.0	1.08	1.07
4	3.3	3.1	2.5	2.3	1.06	1.08

2. The ratios of the side lengths are equal.

3. $\frac{AD}{DC} = \frac{AS}{SR}$

4. The ratio remains the same.

5. The ratio changes when moving point D .

6. When you move the parallel line, you are changing the proportion between the upper and lower segments. When you move the point, the segments may get longer or shorter, but the proportion stays the same.

7. 12

8. 16.8

9. Sample answers:

TABLE 8.2:

Position	RN	NO	EA	AS	$\frac{RN}{NO}$	$\frac{EA}{AS}$
1	2.1	1.5	2.4	1.8	1.4	1.4
2	2.1	1.3	2.7	1.7	1.6	1.6
3	1.9	1.5	2.4	2.0	1.2	1.2
4	2.7	0.7	3.5	0.9	4	4

10. The ratios are equal.

11. $\frac{RN}{NO}$ and $\frac{EA}{AS}$ are congruent

8.3 Perspective Drawing

This activity can be used to supplement the "Know What?" from Chapter 7, Lesson 6.'

The student worksheets for this activity are at the end of Chapter 12 because this activity can also be used at the end of the year. The link is below.

ID: 10033

Time required: 60 minutes

Activity Overview

In this activity, students draw figures in one- and two-point perspective and compare and contrast the two types of drawings. They then create an isometric drawing and compare it to their drawings in perspective.

Topic: 3-Dimensional Geometry

- Construct 3- dimensional prisms and pyramids.
- Record the number of faces, edges, and vertices of prisms.

Teacher Preparation and Notes

- Perspective drawings can be taught at any time in a geometry curriculum, but are most appropriate after lessons on parallel and perpendicular lines, three-dimensional figures, and symmetry.
- Throughout the activity, students use many drawing and construction tools, such as the **Segment, Parallel, and Perpendicular** tools. In this document, the first use of a tool is by name and accompanied by its location within the menu structure. For subsequent uses, the tool may be mentioned by name or its function, and the menu location is omitted.
- This activity is designed to have students explore **individually or in pairs**.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1003> and select **RECPRISM, TRIPRISM, TWOPERSP, ISODRAW**.

Associated Materials

- Student Worksheet: Perspective Drawing <http://www.ck12.org/flexr/chapter/9697>, scroll down to the third activity.
- Cabri Jr. Application
- RECPRISM.8xv, TRIPRISM.8xv, TWOPERSP.8xv, and ISODRAW.8xv

Problem 1 – One-point perspective

Students will open the *Cabri Jr.* file *RECPRISM* and find a vanishing point *A* along the horizon and a rectangle *BCDE*

They are to use the **Segment** tool from the $F2$ menu to draw \overline{AB} and \overline{AC} . These two segments (and any other segment that joins a vertex of the prism to the vanishing point) are hereafter referred to as *vanishing segments*.

Next, they will create \overline{FG} such that F is between A and B , G is between A and C , and $\overline{FG} \times \overline{BC}$ (Refer to the diagram at the right).

To do this, they will need to do the following:

- Place a point F on \overline{AB} . (**F2 #62; Point #62; Point On**)

Construct a line through F parallel to \overline{BC} .

(**F3 #62; Parallel**)

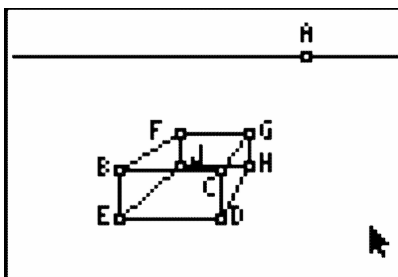
- Plot point G at the intersection of the parallel line and \overline{AC} . (**F2 #62; Point #62; Intersection**)
- Hide the line. (**F5 #62; Hide/Show #62; Object**)
- Draw \overline{FG} .

Students can now hide the vanishing segments, \overline{AB} and \overline{AC} , and draw \overline{BF} and \overline{CG} . The front and top faces of the rectangular prism have been constructed.

Ask students if they are satisfied that this drawing completely represents a rectangular prism. Have them drag point A (press **APLHA** to grab the point) to either the far left or far right and ask students what is wrong with the drawing. (The rectangular prism does not have a face on either side.)

To complete the prism, students next need to construct \overline{AD} and \overline{AE} . Then, using the **Perpendicular** tool (**F3 #62; Perp.**), construct two lines perpendicular to \overline{FG} —one through F and the other through G . The vertical “back” edges \overline{GH} and \overline{FJ} may then be created by plotting the intersection points, hiding the lines, and constructing the segments as needed.

To show all six edges, students should draw a segment connecting H and J . Once all of the vanishing segments are hidden, students will find they need to also construct segments \overline{DH} and \overline{EJ} .



Depending on the topics you have recently covered in class, you can now have students identify parallel, intersecting, and skew lines; count faces, vertices, and edges; or discuss the similarity of rectangles $BCDE$ and $FGHJ$.

When students have finished exploring their prisms, they may wish to hide or dash the edges that would not be visible if the figure were not transparent. The appearance of the edges may be altered using the **Display** tool (**F5 #62; Display**).

Students are to open **TRIPRISM** and create a triangular prism in one-point perspective on their own. Be sure to circulate around the room and assist students as needed.

Allow students a few minutes to explore the figure by dragging point A along the horizon or dragging an edge. Vertices B , C , and D can also be dragged on this figure.

Problem 2 – Two-point perspective

The Cabri Jr. file **TWOPERSP** contains two vanishing points, A and B , signifying this rectangular prism will be drawn in two-point perspective. The file also contains a vertical line segment, \overline{CD} , which will be the front edge of the prism.

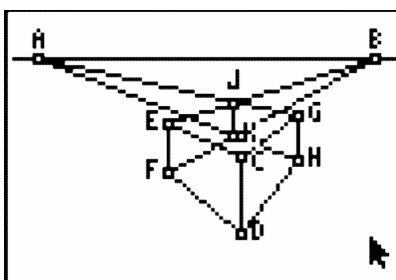
\overline{CD} is constructed so that dragging point C changes the segment's length and dragging point D translates the segment.

Next, students will then use the **Segment** tool to draw the vanishing segments \overline{AC} , \overline{AD} , \overline{BC} , and \overline{BD} . Then they should use the **Point On, Parallel, Intersection, Hide/Show**, and **Segment** tools (as they did in Problem 1) to construct the vertical edges, \overline{EF} and \overline{GH} , of the left and right faces.

The top and bottom edges of those faces, \overline{CE} , \overline{CG} , \overline{DF} , and \overline{DH} may be drawn as well.

Students should now hide the four vanishing segments, and then draw four more: from E and F to B and from G and H to A .

The intersection of the upper two vanishing segments and that of the lower two vanishing segments should be plotted and a segment drawn between them to form the last vertical edge, \overline{JK} .



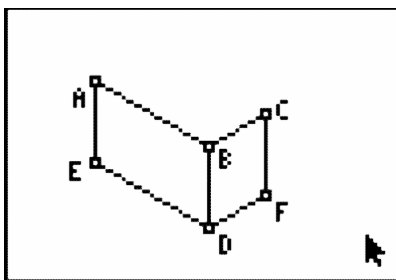
The vanishing segments may then be hidden so that students can draw the remaining edges of the prism.

As before, allow students some time to drag edges, vertices, and vanishing points, observing how the prism changes as they do so. (Dragging \overline{CD} is very interesting!) Again, students can either hide or dash any “hidden edges.”

Problem 3 – An isometric drawing

Using **ISODRAW**, students will construct an isometric drawing of a rectangular prism. The three segments shown represent the three “front” edges of the prism, and each of the angles the segments form measures 120° . Students should first use the **Parallel, Intersection, Hide/Show**, and **Segment** tools to construct \overline{AE} , \overline{CF} , \overline{DE} , and \overline{DF} , as shown in the diagram to the right.

They may then use the same sequence of tools once more to construct the remaining edges: \overline{AG} , \overline{CG} , \overline{EH} , \overline{FH} , and \overline{GH} .



After completing the isometric drawing, students should compare it to the perspective drawings from Problems 1 and 2. As a similarity, students may identify that right angles “in real-life” do not actually measure 90° in the drawings. As a difference, all parallel lines “in real-life” are parallel in an isometric drawing, whereas in perspective drawings, *some* parallel lines actually “meet” at the vanishing point.

CHAPTER 9**TE Right Triangle
Trigonometry - TI****CHAPTER OUTLINE**

- 9.1 THE PYTHAGOREAN THEOREM**
 - 9.2 INVESTIGATING SPECIAL TRIANGLES**
 - 9.3 RATIOS OF RIGHT TRIANGLES**
-

9.1 The Pythagorean Theorem

This activity is intended to supplement Geometry, Chapter 8, Lesson 1.

ID: 9532

Time required: 60 minutes

Activity Overview

In this activity, students will use the Cabri Jr. application to construct figures that prove the Pythagorean Theorem in two different ways.

Topic: Right Triangles #38; Trigonometric Ratios

- *Construct and measure the side lengths of several right triangles and conjecture a relationship between the areas of the squares drawn on each side.*
- *Prove and apply the Pythagorean Theorem.*

Teacher Preparation

- *This activity is designed to be used in a high school or middle school geometry classroom.*
- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress. **Feel free to print out the following pages for your students.***
- *The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs. This can be expressed as $c^2 = a^2 + b^2$ where c is the length of the hypotenuse.*
- *Depending on student skill level, you may wish to download the constructed figures to student calculators. If the files are downloaded, skip the construction steps for each problem and begin each at **Step 10**.*
- ***Note:** Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:*

1. Move the cursor over the value so it is highlighted.
2. Press + to display additional decimal digits or - to hide digits.

- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=9532> and select **PYTHAG1** and **PYTHAG2**.*

Associated Materials

- Student Worksheet: The Pythagorean Theorem <http://www.ck12.org/flexr/chapter/9693>
- Cabri Jr. Application
- PYTHAG1.8xv and PYTHAG2.8xv

Problem 1 – Squares on Sides Proof

The Pythagorean Theorem states that, the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. In this activity, you will construct a right triangle and verify the Pythagorean Theorem

by constructing squares on each side and comparing the sum of the area of the two smaller squares to the area of the square of the third side.

Step 1: Open a new Cabri Jr. file.

Construct a segment using the **Segment** tool.

Select the **Alpha-Num** tool to label the endpoints B and C as shown.

Step 2: Construct a line through C that is perpendicular to \overline{BC} using the **Perp.** tool.

Step 3: Construct a point on the perpendicular line and label it A .

Hide the perpendicular line with the **Hide/Show** tool and construct line segments \overline{AC} and \overline{AB} .

For the time being, keep the sides of the triangle fairly small so that squares can be constructed on the sides.

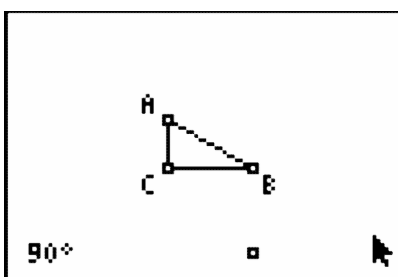
Step 4: In the lower left corner, use the **Alpha-Num** tool to place the number 90 on the screen. This will be the angle of rotation.

Note: Press **ALPHA** to access numerical characters. A small “1” will appear in the tool icon in the upper left corner of the screen.

Step 5: Use the **Rotation** tool to rotate point C about point B through an angle of 90° .

- Press **ENTER** on point B as the center of rotation.
- Press **ENTER** on the angle of rotation (the number 90).
- Press **ENTER** on point C , the object to be rotated.

Notice that the number now has a degree symbol associated with it and that the point has been rotated in the counter-clockwise direction.



Step 6: What we want to do next is to rotate point B about point C through an angle of 90° in the *clockwise* direction. To do this we will need an angle of -90 . Place this number on the screen.

Using the value of -90 , rotate point B about point C through an angle of -90° .

Step 7: You should now have two points below the line segment \overline{BC} . Use the **Quad.** tool to construct a quadrilateral using points B , C and the two points constructed in Steps 5 and 6.

Answer Question 1 on the worksheet.

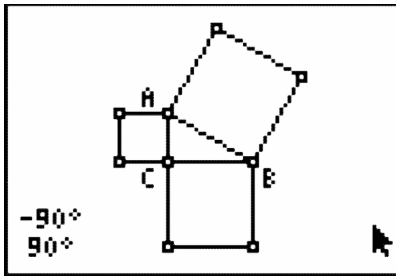
Step 8: In a similar fashion, rotate point C about point A through an angle of -90° and rotate point A about point C through an angle of 90° . This will allow us to construct a second square.

Use the **Quad.** tool again to construct the square on side \overline{AC} .

Step 9: Finally, rotate point B about point A through an angle of 90° and rotate point A about point B through an angle of -90° .

Then construct a third square on hypotenuse \overline{AB} .

9.1. THE PYTHAGOREAN THEOREM



Step 10: Start with this step if you are using the pre-constructed file “PYTHAG1”.

Select the **Measure #62; Area** tool and measure the area of the three squares.

Step 11: Using the **Calculate** tool, press **ENTER** on the measurements of the two smaller squares and then press the + key. Place the sum off to the side of the screen.

How does this sum compare to the square of the hypotenuse? Record your observations in the table for Question 2 on the worksheet.

Step 12: To test your construction, drag points A , B and/or C to a new location on the screen.

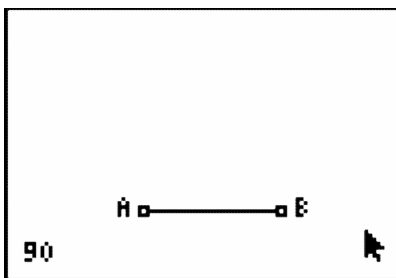
Answer Question 3 on the worksheet.

Problem 2 – Inside a Square Proof

In this problem, we are going to look at a proof of the Pythagorean Theorem. We hope to prove the statement that, if z is the length of the hypotenuse of a right triangle and x and y are the lengths of the legs of the right triangle, then $z^2 = x^2 + y^2$.

Step 1: Construct a line segment \overline{AB} .

Use the **Alph-Num** tool to place the value 90 on your screen.



Step 2: Access the **Rotation** tool and press **ENTER** on point A as the center of the rotation, then on 90 as the angle of rotation and finally on line segment \overline{AB} as the object to be rotated.

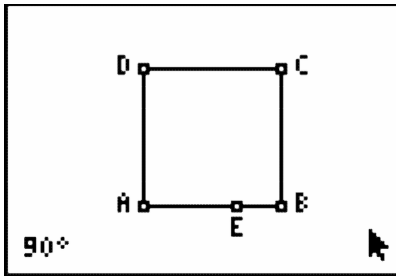
Label the new point D .

Step 3: Continue by rotating line segment \overline{AD} about point D through an angle of 90° .

Label the new point C .

Step 4: Complete the square by constructing line segment \overline{BC} .

Step 5: Using the **Point on** tool, add point E on \overline{AB} as shown and overlay a line segment \overline{BE} .

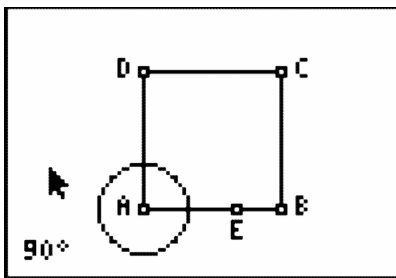


Step 6: Select the **Compass** tool to construct circles with radius equal to the length of \overline{BE} .

- Press **ENTER** on \overline{BE} . A dashed circle will appear and follow the pointer.
- Press **ENTER** on point A . The compass circle is anchored at center A .

Create a point of intersection of this circle with \overline{AD} . Label this point F .

Hide the compass circle.



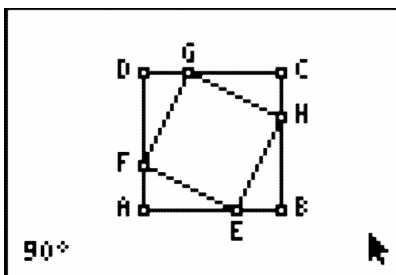
Step 7: Use the **Compass** tool again to construct circles with centers at C and D and radius $= BE$.

Create points of intersection of these circles with \overline{DC} and \overline{BC} . Label these points G and H .

Hide the compass circles.

Drag point E to confirm that F , G , and H all move as E moves.

Step 8: Construct the quadrilateral $EFGH$. Can you prove that this quadrilateral is a square?



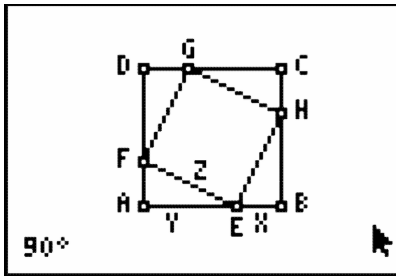
Step 9: Use the **Alph-Num** tool to place the labels x , y , and z on the figure as shown.

\overline{BE} is labeled x , therefore $AF = DG = CH = x$.

\overline{EA} is labeled y , therefore $FD = GC = HB = y$.

Since $ABCD$ is a square, each of the angles at A , B , C and D are 90° , so we have four congruent triangles, namely $\triangle EFA$, $\triangle FGD$, $\triangle GHC$ and $\triangle HEB$.

9.1. THE PYTHAGOREAN THEOREM



Step 10: Start with this step if you are using the pre-constructed file “PYTHAG2”.

Let’s examine the algebra in this situation.

$ABCD$ is a square with sides of length $(x + y)$.

The area of the square $ABCD$ is $(x + y)^2 = x^2 + 2xy + y^2$.

Each of the triangles $\triangle EFA$, $\triangle FGD$, $\triangle GHC$ and $\triangle HEB$ is a right triangle with height x and base y . So, the area of each triangle is $\frac{1}{2}xy$. The sum of the areas of the four triangles is $4 \cdot \frac{1}{2}xy = 2xy$.

$EFGH$ is a square with sides of length z . So the area of $EFGH$ is z^2 .

Looking at the areas in the diagram we can conclude that:

$$ABCD = \triangle EFA + \triangle FGD + \triangle GHC + \triangle HEB + EFGH$$

On the worksheet, substitute the area expressions (with variables x , y , and z) into the equation above and simplify.

Step 11: Let’s look at this numerically as well to confirm what we just proved algebraically. Measure \overline{BE} , \overline{HB} and \overline{EH} .

Note: Measure \overline{HB} and \overline{EH} by pressing **ENTER** on each endpoint, since these do not have separate segments constructed.

Use the **Calculate** tool to find the squares of these lengths.

Record your observations in the table for Question 6 on the worksheet.

Step 12: Find the sum of the squares of the lengths of segments \overline{BE} and \overline{HB} .

In the right triangle HBE , is $BE^2 + HB^2 = EH^2$?

Drag point E to ensure that the relationship holds for other locations of the points E , F , G , and H .

What would happen if you dragged one of points A or B ? Would the relationship still hold?

Answers Questions 7 and 8 on the worksheet.

9.2 Investigating Special Triangles

This activity is intended to supplement Geometry, Chapter 8, Lesson 4.

ID: 7896

Time required: 45 minutes

Activity Overview

In this activity, students will investigate the properties of an isosceles triangle. Then students will construct a $30^\circ - 60^\circ - 90^\circ$ triangle and a $45^\circ - 45^\circ - 90^\circ$ triangle to explore the ratios of the lengths of the sides.

Topic: Right Triangles #38; Trigonometric Ratios

- Calculate the trigonometric ratios for $45^\circ - 45^\circ - 90^\circ$, $60^\circ - 60^\circ - 60^\circ$ and $30^\circ - 60^\circ - 90^\circ$ triangles.

Teacher Preparation and Notes

This activity is designed to be used in a high school or middle school geometry classroom.

- If needed, review or introduce the term median of a triangle. Any median of an equilateral triangle is also an altitude, angle bisector, and perpendicular bisector.
- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively.
- The worksheet guides students through the main ideas of the activity and provides a place for students to record their work. You may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.
- To download **Cabri Jr**, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=7896> and select **EQUIL** and **ISOSC**.

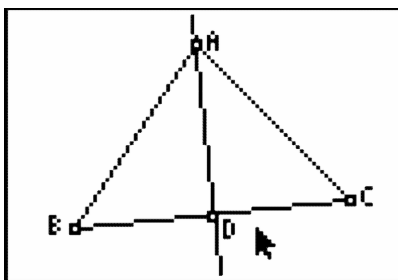
Associated Materials

- Student Worksheet: Investigating Special Right Triangles <http://www.ck12.org/flexr/chapter/9693>, scroll down to the second activity.
- Cabri Jr. Application
- EQUIL.8xv and ISOSC.8xv

Problem 1 – Investigation of Triangles

First, turn on your TI-84 and press **APPS**. Arrow down until you see **Cabri Jr** and press **ENTER**. Open the file **ISOSC**. This file has a triangle with an isosceles triangle with $AB = AC$.

Using the **Perpendicular** tool (**ZOOM #62; Perp.**), construct a perpendicular from point A to side BC . Label the point of intersection of this line with BC as D . To name the point, they need to select the **Alph-Num** tool (**GRAPH #62; Alph-Num**), select the point, and press x^{-1} **ENTER** for the letter D .



Construct line segments BD and CD (**WINDOW #62; Segment**) and then measure the segments (**GRAPH #62; Measure #62; D. #38; Length**).

Would you have expected these segments to be equal in length?

Drag point C to see the effect on the lengths of the line segments. It appears that the perpendicular from the vertex always bisects the opposite side. Measure the angles BAD and CAD .

Will they always be equal?

Problem 2 – Investigation of 30-60-90 Triangles

Open the file **EQUIL**. Note that all three angles are 60° angles.

Construct the perpendicular from A to side BC . Label the point of intersection as D , like in Problem 1.

From the construction above, we know that D bisects BC and that $m\angle BAD = 30^\circ$.

Construct segment BD . We now have triangle BAD where $m\angle D = 90^\circ$, $m\angle B = 60^\circ$ and $m\angle A = 30^\circ$. We also have triangle ACD where $m\angle A = 30^\circ$, $m\angle C = 60^\circ$ and $m\angle D = 90^\circ$.

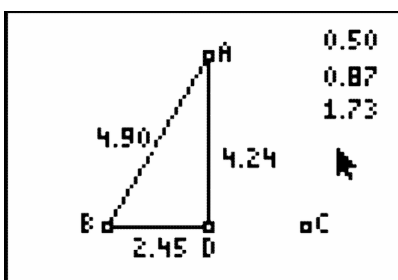
This completes the construction of two $30^\circ - 60^\circ - 90^\circ$ triangles. We will work only with the triangle BAD .

You may choose to have the students hide the segments AC and CD . To do this, construct segments BD and AB on top of the larger triangle. Then hide the original triangle. Keep the point C . We will need that point later to resize the triangle.

Measure the three sides of the triangle.

Press **GRAPH** and select the **Calculate** tool. Click on the length of BD , then on the length of AB . Press the \div key. **Students will see the result 0.5**. Move it to the upper corner. Repeat this step to find the ratio of $AD : AB$ and $AD : BD$. These ratios will become important when you start working with trigonometry.

Drag point C to another location.

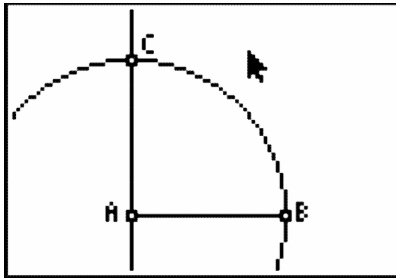


What do you notice about the three ratios?

Problem 3 – Investigation of 45-45-90 Triangles

Press the $Y =$ button and select **New** to open a new document.

To begin the construction of the $45^\circ - 45^\circ - 90^\circ$ triangle, construct line segment AB and a perpendicular to AB at A . Use the compass tool with center A and radius AB . The circle will intersect the perpendicular line at C .



Hide the circle and construct segments AC and BC .

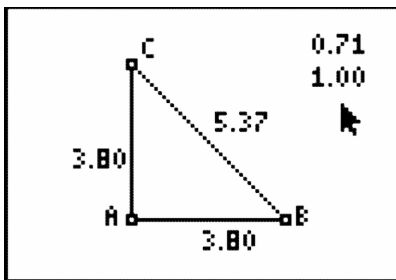
Can you explain why $AB = AC$ and why angle $ACB =$ angle ABC ?

Why are these two angles 45° each?

Students should notice that the two angles must be equal, and angle A is 90° . Therefore, because the sum of the angles in a triangle is 180° , the two angles must be 45° each.

Measure the sides of the triangle. **This verifies that $AB = AC$.**

Use the **CALCULATE** tool to find the ratio of $AC : BC$ and $AC : AB$. Once again, these ratios will be important when you study trigonometry.



Drag point B and observe what happens to the sides and ratios.

Why do the ratios remain constant while the sides change?

Students should notice that AC and AB are equal, so the ratios will always remain the same.

9.3 Ratios of Right Triangles

This activity is intended to supplement Geometry, Chapter 8, Lesson 5.

ID: 11576

Time Required: 45 minutes

Activity Overview

In this activity, students will explore the ratios of right triangles. Students will discover that they can find the measure of the angles of a right triangle given the length of any two sides.

Topic: Right Triangles #38; Trigonometric Ratios

- Sine
- Cosine
- Tangent

Teacher Preparation and Notes

- *This activity was written to be explored on the TI-84 with the Cabri Jr. and Learning Check applications.*
- *Before beginning this activity, make sure that all students have the Cabri Jr. applications. Also, make sure students have or know the trigonometric definitions.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=11576> and select TRIG.*

Associated Materials

- Student Worksheet: Ratios of Right Triangles <http://www.ck12.org/flexr/chapter/9693>, scroll down to the third activity.
- Cabri Jr. Application
- TRIG.8xv

Problem 1 – Exploring Right Triangle Trigonometry

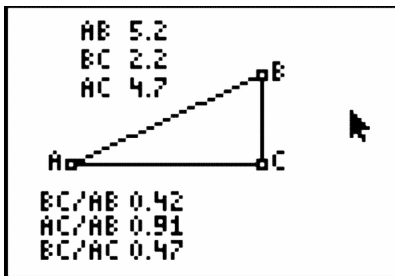
You may need to allow students to use a textbook (or other resource) to find the definitions of sine, cosine, and tangent.

Students are asked to give the ratio of several triangles on their handheld or their accompanying worksheet.

Problem 2 – Exploring the Sine Ratio of a Right Triangle

For this problem, students will investigate the sine ratio of two sides of a triangle. Students should start the *Cabri Jr.* app and open the file *Trig.8xv*.

Students will collect data on their worksheets by moving point B . They will do this for four different positions of the point.



Students will discover that the ratio of BC to AB remains constant, no matter how large the triangle is; Therefore, students will be able to use the inverse of sine to find the measure of the angles in $\triangle ABC$.

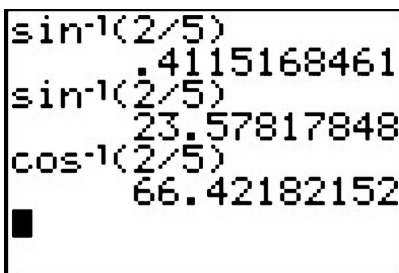
Students will need to answer several questions on their handhelds or their accompanying worksheets.

Problem 3 – Exploring the Cosine Ratio of a Right Triangle

Students will repeat the exploration in Problem 2, but with the cosine ratio.

Problem 4 – Applying the Sine, Cosine, and Tangent Ratio of a Right Triangle

In Problem 4, students are asked to apply what they have learned about how to find the measure of an angle of a right triangle given two sides of the triangle.



Solutions

1. For right triangle ABC , the sine of an angle is the ratio of the length of the opposite side to the length of the hypotenuse.

2. For right triangle ABC , the cosine of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse.
3. For right triangle ABC , the tangent of an angle is the ratio of the length of the opposite side to the length of the adjacent side.
4. $\frac{3}{5}$
5. $\frac{4}{5}$
6. $\frac{3}{4}$
7. $\frac{4}{5}$
8. $\frac{3}{5}$
9. $\frac{4}{3}$

10. Sample answers:

TABLE 9.1:

Position	BC	AB	$\frac{BC}{AB}$	$\sin^{-1} \frac{BC}{AB}$
1	2.4376781463393	6.2006451991814	0.39313298342907	23.149583787224
2	3.0769811671077	7.8268201774092	0.39313298342907	23.149583787224
3	3.6665092204124	9.3263841370716	0.39313298342906	23.149583787223
4	4.3154341679767	10.977034107736	0.39313298342905	23.149583787222

11. The ratio does not change.

12. No, the angle does not change.

13. 23.1496

14. 66.8504

15. Sample answers:

TABLE 9.2:

Position	AC	AB	$\frac{AC}{AB}$	$\cos^{-1} \frac{AC}{AB}$
1	7.0816099136391	8.8549125969341	0.7997379800328	36.894911430193
2	8.0238624186986	10.033114118664	0.79973798003277	36.894911430196
3	9.0235078139592	11.283080257848	0.79973798003279	36.894911430194
4	3.7704816328074	4.7146462053143	0.79973798003281	36.894911430192

16. 36.8949

17. 53.1051

18. $A = \tan^{-1} \frac{BC}{AC}$

19. $A = 23.57, B = 66.42$

20. $A = 21.8, B = 68.2$

21. $A = 23.96, B = 66.04$

22. $A = 53.13, B = 36.87$

23. $A = 15.07, B = 74.93$

24. $A = 42.83, B = 47.17$

25. $A = 45, B = 45$

26. $A = 29.05, B = 60.95$

CHAPTER 10**TE Circles - TI****CHAPTER OUTLINE**

10.1 CHORDS AND CIRCLES**10.2 INSCRIBED ANGLES THEOREM****10.3 CIRCLE PRODUCT THEOREMS**

10.1 Chords and Circles

This activity is intended to supplement Geometry, Chapter 9, Lesson 3.

ID: 9773

Time required: 30 minutes

Topic: Circles

- Deduce from the Perpendicular Bisector Theorem the following corollaries:

- a) The perpendicular from the center of a circle to a chord bisects the chord.
- b) The line joining the center of a circle to the midpoint of a chord is perpendicular to the chord.
- c) The center of a circle is at the intersection of the perpendicular bisector of two non-parallel chords.

Activity Overview

Students will begin this activity by exploring how the chord in a circle is related to its perpendicular bisector. Investigation will include measuring lengths and distances from the center of the circle. These measurements will then be transferred to a graph to see the locus of the intersection point of the measurements as the endpoint of a chord is moved around the circle. In the extension, students will be asked to find an equation for the ellipse that models the relationship.

Teacher Preparation

This activity is designed to be used in a high school geometry classroom.

- Students should already be familiar with circles, chords of circles, and perpendicular bisectors.

Classroom Management

- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.
- Information for an optional extension is provided at the end of this activity.
- To download *Cabri Jr*, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.

Associated Materials

- Student Worksheet: Chords and Circles <http://www.ck12.org/flexr/chapter/9694>
- Cabri Jr. Application

Introduction

When hikers and skiers go into terrain where there is a risk of avalanches, they take safety equipment including avalanche rescue beacons. An avalanche rescue beacon sends and receives electromagnetic field signals that can travel up to about 30 meters. The search pattern used to locate a beacon buried in the snow is based on the properties of chords and diameters in circles. In this activity, you will use Cabri Jr. to model an avalanche search pattern.

Problem 1 – Relationship between a chord and its perpendicular bisector

Construct a circle using the **Circle** (*F2 #62;Circle*) tool in Cabri Jr. to represent the beacon signal.

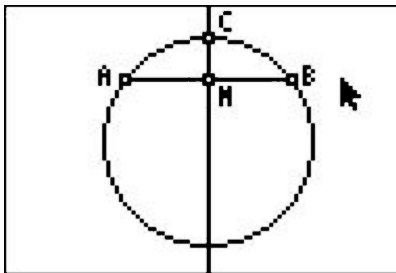
Use the **Hide/Show** to hide its center.

Construct a chord to represent the path of a rescuer as he walks a straight path until the signal disappears. Use the **Segment** tool to draw the chord with endpoints on the circle and label it AB . [U+F020]

The rescuer walks back to the midpoint of this path. Find the midpoint of AB and label it M .

Construct a line perpendicular to AB through M , to represent the rescuer walking away from the path at a 90° angle until the signal disappears.

Find one intersection point of the perpendicular line and the circle. Label it C .



The rescuer turns around and walks in the opposite direction until the signal disappears again. Find the other intersection point of the perpendicular line and the circle. Label it D .

Hide the perpendicular line. Construct a segment connecting points C and D .

The rescuer walks back to the midpoint of this new path.

Find the midpoint of CD and label it X . This will be the center of the circle formed by the beacon signals. Dig for the missing person!

Confirm that you have located the center of the circle. Measure the distances from X to A , B , C , and D .

Problem 2 – Extension

Extension 1

Write a proof of the relationship used in the activity. Given a chord of a circle and its perpendicular bisector, prove that the perpendicular bisector passes through the center of the circle.

Extension 2

Use a compass and straightedge to construct a circle and a chord. Construct the perpendicular bisector of the chord and see that it passes through the center of the circle.

10.2 Inscribed Angles Theorem

This activity is intended to supplement Geometry, Chapter 9, Lesson 4.

ID: 12437

Time Required: 15 minutes

Activity Overview

Students will begin this activity by looking at inscribed angles and central angles and work towards discovering a relationship among the two, the Inscribed Angle Theorem. Then, students will look at two corollaries to the theorem.

Topic: Circles

- Construct central and inscribed angles
- Inscribed angles theorem

Teacher Preparation and Notes

- This activity was written to be explored with the Cabri Jr. application on the TI-84.
- To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.
- To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1243> and select INSCRIB1-INSCRIB5.

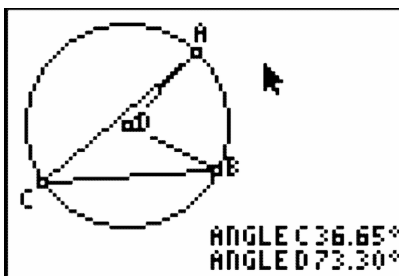
Associated Materials

- Student Worksheet: Inscribed Angle Theorem <http://www.ck12.org/flexr/chapter/9694>, scroll down to the second activity.
- Cabri Jr. Application
- INSCRIB1.8xv, INSCRIB2.8xv, INSCRIB3.8xv, INSCRIB4.8xv, and INSCRIB5.8xv

Problem 1 – Similar Triangles

Students will begin this activity by looking at inscribed angles and central angles and work towards discovering a relationship among the two.

Students will be asked to collect data by moving points A and C . Students are asked questions about the relationships in the circle and are asked to make a conjecture. In order to calculate the ratio of $m\angle ACB$ to $m\angle ADB$, students can use the **Calculate** tool in the Actions menu.

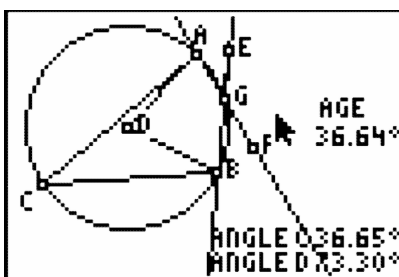


On page 1.6, students will look at two inscribed angles intercepted by the same arc and are asked to make a conjecture about the relationship.

In an advanced setting a proof of the inscribed angle theorem and the two conjectures in problem one are appropriate and can be proved using isosceles triangles.

Problem 2 – Extension of the Inscribed Angle Theorem

In Problem 2, students will look at two more angles created from the central angle and the intercepted arc. Both sections of this problem are corollaries of the Inscribed Angle Theorem and both solutions are congruent to the measure of the central angle intercepted by the arc or one-half the measure of the central angle.



Solutions

1. Sample answers

TABLE 10.1:

Position	Measure of $\angle ACB$	Measure of $\angle ADB$	$\frac{m\angle ACB}{m\angle ADB}$
1	36.65°	73.30°	0.5
2	49.50°	99.01°	0.5
3	49.50°	99.01°	0.5
4	49.50°	99.01°	0.5

2. $\frac{1}{2}$

3. Sample answers

TABLE 10.2:

Position	Measure of $\angle ACB$	Measure of $\angle AEB$
1	36.65°	36.65°
2	48.30°	48.30°
3	48.30°	48.30°
4	48.30°	48.30°

4. Sample answer: They are congruent.

5. diameter of the circle

6. 90°

7. Sample answers

TABLE 10.3:

Position	Measure of $\angle ACB$	Measure of $\angle ADB$	Measure of $\angle AGE$
1	36.65°	73.30°	36.65°
2	67.04°	134.10°	67.04°
3	67.04°	134.10°	67.04°
4	67.04°	134.10°	67.04°

8. $\frac{1}{2}$

9. Sample answers

TABLE 10.4:

Position	Measure of $\angle ACB$	Measure of $\angle ADB$	Measure of $\angle ABE$
1	57.65°	115.30°	57.64°
2	49.20°	98.40°	49.19°
3	54.90°	109.80°	54.90°
4	63.55°	127.10°	63.54°

10. $\frac{1}{2}$

10.3 Circle Product Theorems

This activity is intended to supplement Geometry, Chapter 9, Lesson 6.

ID: 12512

Time Required: 20 minutes

Activity Overview

Students will use dynamic models to find patterns. These patterns are the Chord-Chord, Secant-Secant, and Secant-Tangent Theorems.

Topic: Circles

- *Chord-Chord, Secant-Secant, and the Secant-Tangent Product Theorems*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. application on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1251> and select PRODUC1-PRODUC5.*

Associated Materials

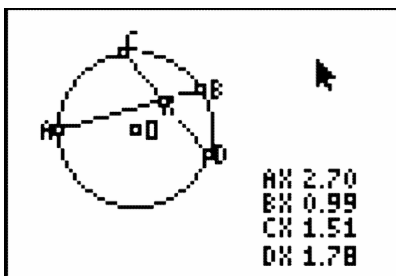
- Student Worksheet: Circle Product Theorems <http://www.ck12.org/flexr/chapter/9694>, scroll down to the third activity.
- Cabri Jr. Application
- PRODUC1.8xv, PRODUC2.8xv, and PRODUC3.8xv

Problem 1 – Chord-Chord Product Theorem

Students will begin this activity by investigating the intersection of two chords and the product of the length of the segments of one chord and the product of the length of the segments of the other chord.

Students will be asked to collect data by moving point A . Students are asked to calculate the products by hand on their accompanying worksheet. Students are asked several questions about the relationship among the products.

As an extension, prove the chord-chord product theorem using similar triangles.

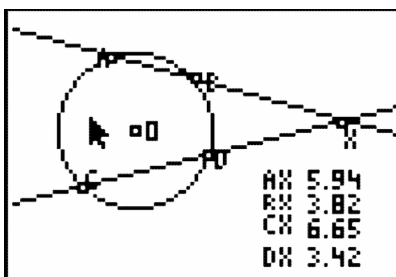


Problem 2 – Secant-Secant Product Theorem

Students will investigate the intersection of two secants and the product of the lengths of one secant segment and its external segment and the product of the lengths of the other secant segment and its external segment.

Students will be asked to collect data by moving point A . They are to calculate the products by hand on their accompanying worksheet. Students are asked several questions about the relationship among the products.

As an extension, prove the secant-secant product theorem using similar triangles.

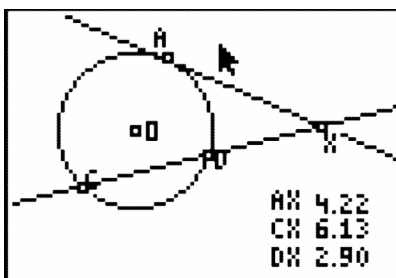


Problem 3 – Secant-Tangent Product Theorem

Students will investigate the intersection of the product of the lengths of one secant segment and its external segment and the square of the tangent segment.

Students will be asked to collect data by moving point A . Students are asked to calculate the products by hand on their accompanying worksheet. Students are asked several questions about the relationship among the products.

As an extension, prove the secant-tangent product theorem using similar triangles.



Problem 4 – Application of the Product Theorems

Students will be asked to apply what they learned in Problems 1–3 to solve a few problems.

Solutions

1. Sample answers:

TABLE 10.5:

Position	AX	BX	CX	DX	$AX \cdot BX$	$CX \cdot DX$
1	2.70	0.99	1.51	1.78	2.67	2.68
2	2.51	1.05	1.38	1.90	2.63	2.62
3	2.93	0.91	1.80	1.49	2.67	2.68
4	1.80	1.08	2.51	0.77	1.94	1.93

2. They are equal.

3. equal

4. Sample answers:

TABLE 10.6:

Position	AX	BX	CX	DX	$AX \cdot BX$	$CX \cdot DX$
1	5.94	3.82	6.65	3.42	22.69	22.74
2	4.85	3.16	5.85	2.62	15.33	15.33
3	4.03	2.75	5.32	2.09	11.08	11.12
4	7.47	4.96	7.92	4.68	37.05	37.07

5. They are equal.

6. equals

7. Sample answers:

TABLE 10.7:

Position	AX	CX	DX	AX^2	$CX \cdot DX$
1	4.22	6.13	2.90	17.81	17.77
2	3.42	5.40	2.16	11.70	11.66
3	2.45	4.56	1.32	6.00	6.02
4	9.46	11.21	7.98	89.49	89.46

8. They are equal.

9. equals

10. 6

11. $\frac{3}{4}$

12. $3\sqrt{13}$ or 10.817

CHAPTER

11**TE Perimeter and Area - TI****CHAPTER OUTLINE**

11.1 DIAMETER AND CIRCUMFERENCE OF A CIRCLE

11.2 FROM THE CENTER OF THE POLYGON

11.1 Diameter and Circumference of a Circle

This activity is intended to supplement Geometry, Chapter 10, Lesson 4.

ID: 9844

Time required: 30 minutes

Topic: Circles

- *Use technology to verify the circumference and area formulas for the circle.*

Activity Overview

In this activity, students explore the relationship between a circle's circumference and its diameter. This will lead students to their own discovery of a value for pi.

Teacher Preparation

This activity is designed to be used in a high school geometry classroom.

- *Students should already be familiar with circles, diameter, circumference, and pi.*

Classroom Management

- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*

Associated Materials

- Student Worksheet: Diameter and Circumference of a Circle <http://www.ck12.org/flexr/chapter/9695>
- Cabri Jr. Application

In this activity we will

- *Draw a circle*
- *Measure the diameter of the circle*
- *Measure the circumference of the circle*
- *Calculate the ratio of the circumference to the diameter.*

Problem 1

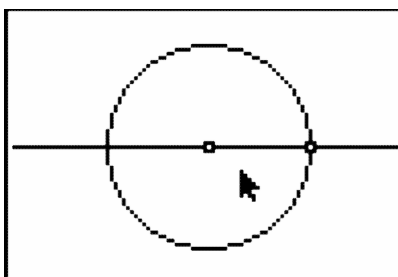
Press **APPS**. Move down to the Cabri Jr. **APP** and press **ENTER**. Press **ENTER**, or any key, to begin using the application.

Press **Y =** for the **F1** menu and select **New**.

(If asked to **Save changes?** press ← **ENTER** to choose “No.”)

Press **WINDOW** for the *F2* menu, move down to **Circle**, and press **ENTER**. Press **ENTER** to mark the center of the circle, then move the pencil to indicate the length of the radius, and press **ENTER** to complete the circle.

Draw a line through the two points which determined the circle. To do this, press **WINDOW** for the *F2* menu, move to **Line**, then press **ENTER**. Move the pencil until the point on the circle is flashing, and press **ENTER**. Now move the pencil until the center of the circle is flashing, and press **ENTER**. Press **CLEAR** to exit the line drawing tool.



Press **WINDOW** for *F2* and move to **Point**. Move to the right and down to select **Intersection**. Press **ENTER**. Move the pencil until both the line and the circle are flashing. Press **ENTER** to mark the point which is the intersection of the circle and the line. Now we have two points on the circle which are the endpoints of a diameter

To measure the circle's diameter, press **GRAPH** for *F5* and move down and right to select **Measure, D. #38; Length**. Press **ENTER**.

Move the pencil until one endpoint of the diameter is flashing then press **ENTER**. Move to the other endpoint of the diameter and when it is flashing, press **ENTER**. Press + to see the measurement rounded to hundredths. The *hand* is active so you can move the measurement to a convenient location then press **ENTER**.

The **Measurement** tool is still active so now you can find the circumference of the circle. Move the pencil until the circle is flashing. Press **ENTER** then + to see the circumference rounded to hundredths. Move the *hand* until the measurement is in a convenient location. Press **ENTER**. Press **CLEAR** to turn off the measurement tool.

Press **GRAPH** for *F5* and move down to **Calculate**. Press **ENTER**. Move the arrow until the circumference measurement shows a flashing underline and press **ENTER** then \div . Move the arrow until the diameter measurement has a flashing underline and press **ENTER** again. The number displayed is the ratio of the circle's circumference to its diameter.

To explore this relationship with other circles, press **CLEAR** to turn off the **Calculate** tool. Move the arrow until the point which defined the circle's radius or its center is flashing. Press **ALPHA** to activate the *hand*. Grab the point and move it to change the size of the circle.

To confirm that the ratio is still 3.14, repeat the **Calculate** procedure. (It is actually being recalculated each time the circle changes, but it is impossible to tell this since the number is unchanging.)

To exit the **APP**, press $Y =$ for the *F1* menu. Move to **Quit**, then press **ENTER**.

11.2 From the Center of the Polygon

This activity is intended to supplement Geometry, Chapter 10, Lesson 6.

ID: 11644

Time Required: 45 minutes

Activity Overview

Students will explore the area of a regular polygon in terms of the apothem and the perimeter. They will derive the formula for a regular pentagon and regular hexagon. Then, students will see how the formula relates to the formula for the area of triangles. Students will then be asked to apply what they have learned about the area of a regular polygon.

Topic: Quadrilaterals #38; General Polygons

- *Regular Polygons*
- *Area of Regular Polygons*

Teacher Preparation and Notes

- *This activity was written to be explored with the Cabri Jr. app on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1164> and select PENTAGON, HEXAGON, OCTAGON.*

Associated Materials

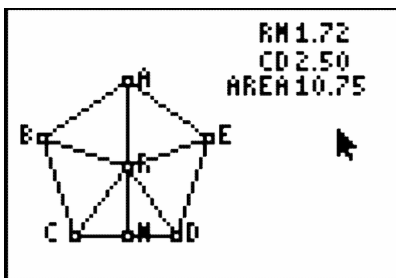
- Student Worksheet: From the Center of a Polygon <http://www.ck12.org/flexr/chapter/9695>, scroll down to the second activity.
- Cabri Jr. Application
- PENTAGON.8xv, HEXAGON.8xv, OCTAGON.8xv

Problem 1 – Area of a Regular Pentagon

Students will begin this activity by looking at a regular pentagon. In file *PENTAGON*, students are given regular pentagon $ABCDE$ with center R . Students are given the length of \overline{CD} (side of the polygon), \overline{RM} (apothem), and the area of the polygon. Students are to collect data in the table given on their student worksheet and contains 4 columns: **Apothem**, **Perimeter**, $a \cdot p$ (**apothem times perimeter**), and **Area**.

Students will collect data by moving point D . They will do this for four different positions of the point.

Students are asked about how the area and the apothem times the perimeter are related.



Problem 2 – Area of a Regular Hexagon

Students will repeat the same process as Problem 1 for a regular hexagon. Students will begin to discover the formula for a regular polygon is one-half the perimeter times the apothem.

Problem 3 – Area of a Regular Polygon

In this problem, students are to “prove” the formula for the area of a regular polygon by looking at an octagon and the triangles created by the radii of the octagon. If students are confused by the term radius of the polygon, explain that this is the radius of a circle circumscribed about a regular polygon.

Problem 4 – Area of Regular Polygons

In Problem 4, students are asked to apply what they have learned about the area of regular polygons. The students are given a question on the area of a regular polygon and a calculator on each page. The students are to use the calculator to find the area.

Solutions

1. Sample answers:

TABLE 11.1:

Position	Apothem (a)	Perimeter (p)	$a \cdot p$ (apothem times perimeter)	Area
1	1.72	12.5	21.5	10.75
2	1.93	14	27.02	13.49
3	2.13	15.5	33.015	16.53
4	2.55	18.5	47.175	23.55

2. $\text{area} = \frac{1}{2} (\text{apothem})(\text{perimeter}) = \frac{1}{2}ap$

3. Sample answers:

TABLE 11.2:

Position	Apothem (a)	Perimeter (p)	$a \cdot p$ (apothem times perimeter)	Area
1	1.91	13.2	25.2	12.6
2	2.25	15.6	35.1	17.76
3	2.34	16.2	37.908	18.9
4	2.60	18	46.8	23.4

4. Area = $\frac{1}{2}$ (apothem)(perimeter) = $\frac{1}{2}ap$

5. 10

6. Yes

7. Yes

8. Area $CDR = \frac{1}{2}as$

9. Area = $\frac{1}{2}as(8) = 4as$

10. Area = $\frac{1}{2}asn$

11. 485.52 sq. in.

12. 25.2 sq. cm

13. 501.84 sq. ft

14. 1,039.2 sq. mm

CHAPTER **12** **TE Surface Area and Volume -**
TI

CHAPTER OUTLINE

12.1 SURFACE AREA OF A CYLINDER

12.1 Surface Area of a Cylinder

This activity is intended to supplement Geometry, Chapter 11, Lesson 2.

ID: 10075

Time required: 30 minutes

Activity Overview

Students will explore a net representation for a right cylinder. The surface area will be developed from the parts of the net.

Topic: 3-Dimensional Geometry

- *Construct 3- dimensional prisms and pyramids from nets.*
- *Calculate the surface area of a right prism or cylinder.*

Teacher Preparation and Notes

- *This activity is designed to be used in a high school or middle school geometry classroom.*
- *This activity is designed to be student-centered.*
- *The surface area of a right cylinder with base radius = R and height = H is $SA = 2\pi R^2 + 2\pi RH$. The activity asks students to notice that the circumference of the circle is the length of the rectangle in the net.*
- *The points R and H control the radius and the height of the cylinder. When R is dragged, the length of the rectangle also changes (because the length = circumference of the circle). The height of the rectangle does not change when R is dragged.*
- **Note:** *Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:*

1. *Move the cursor over the value so it is highlighted.*
2. *Press + to display additional decimal digits or - to decrease digits.*

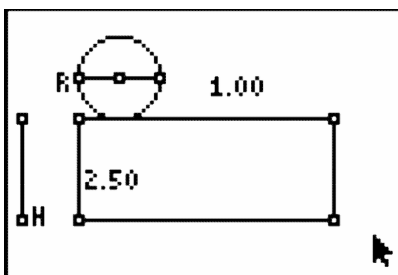
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=10075> and select **CYLINDER**.*

Associated Materials

- Student Worksheet: Surface Area of Cylinders <http://www.ck12.org/flexr/chapter/9696>
- Cabri Jr. Application
- CYLINDER.8xv

Problem 1 – Nets

A net is a pattern that can be cut out and folded into a 3-dimensional figure. Students should see a partial net of a right cylinder. If the rectangle of the net were rolled up, the circle would be the top face of the cylinder (like a lid of a jar).



The dimensions of the net can be changed by dragging the points R and H . Students should drag these points and notice what changes with the figure for each point. When R is moved, the radius of the circle and the width of the rectangle are changed.

Students should use the **D. #38; Length** tool (*F5 #62; Measure*) to find the length of the rectangle.

Note: Display measurements with 2 decimal digits. To do this, hover the cursor over the measurement and then press the plus key (+).

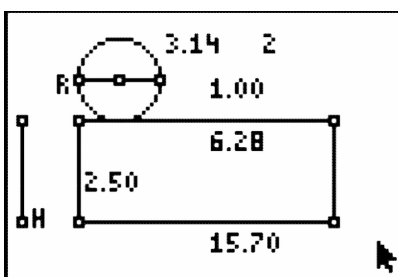
Next, they should use the **Calculate** tool to divide the length of the rectangle by the radius of the circle to find that the width of the rectangle is the same as the circumference of the circle.

Problem 2 – Surface Area

Students should use the **Area** tool from the Measure menu (*F5 #62; Measure*) to find the areas of the rectangle and the circle. Then, they should use the **Alph-Num** tool to place the number 2 on the screen.

Note: Press the **ALPHA** button to access numerical characters. The tool icon in the corner of the screen will display 1A .

Press the **ENTER** button to start and end the text.



Finally, students are to find the surface area of the cylinder. Remind students that this is only a partial net, so one of the faces is missing. To find the surface area, students need to find the area of the circle and the area of the rectangle. They should first use the **Area** tool from the Measurement menu to calculate these areas. Then, they need to use the **Calculate** tool to find the sum of both circle bases by clicking on the area of the circle, pressing \downarrow , and clicking on the 2. Next, they should click on the number they just calculated, press +, and click on the area of the rectangle.

CHAPTER 13**TE Transformations - TI****CHAPTER OUTLINE**

13.1 TRANSFORMATIONS WITH LISTS**13.2 REFLECTIONS AND ROTATIONS****13.3 PERSPECTIVE DRAWING**

13.1 Transformations with Lists

This activity is intended to supplement Geometry, Chapter 12, Lessons 2 and 3.

ID: 10277

Time required: 40 minutes

Activity Overview

Students will graph a figure in the coordinate plane. They will use list operations to perform reflections, rotations, translations and dilations on the figure, and graph the resulting image using a scatter plot.

Topic: Transformational Geometry

- *Perform reflections, rotations, translations and dilations using lists and scatter plots to represent figures on a coordinate plane.*

Teacher Preparation and Notes

- *This activity is designed to be used in a high school geometry or algebra classroom.*
- *If an original point on the coordinate plane is denoted by (x,y) , then each of the following ordered pairs denotes a transformation:*

$(x, -y)$ reflect over x – axis

$(-y,x)$ rotate 90° around origin

$(-x,y)$ reflect over y – axis

$(-x,-y)$ rotate 180° around origin

(y,x) reflect over $y = x$

$(y, -x)$ rotate 90° around origin

- *To perform a translation, add or subtract a constant from the list with the x – values or the y – values of the figure.*
- *To perform a dilation, multiply a constant scale factor by the list with the x – values or the y – values of the figure.*
- *This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively. If desired, have students work in groups of 3. Each person in the group should enter a different combination of lists for Problem 2 and the group should discuss the results.*

Associated Materials

- Student Worksheet: Translations with Lists <http://www.ck12.org/flexr/chapter/9697>

Problem 1 – Creating a Scatter Plot

Before beginning the activity, students need to clear all entries from the $Y =$ screen and all lists.

First, students will enter the data on the worksheet into lists L_1 and L_2 .

After setting up **Plot1** for a scatter plot of L_1 vs L_2 and changing the window settings, students view the graph and sketch the figure on the worksheet. *The shape should be an arrow, in the first quadrant, pointing to the right.*

Problem 2 – Reflections and Rotations

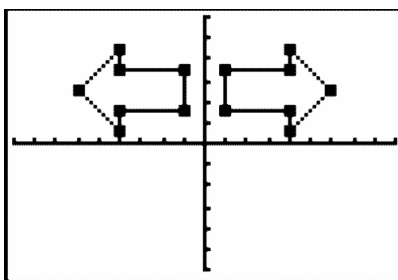
In the list editor students are to enter the formulas $= -L_1$ and $= -L_2$ for L_3 and L_4 respectively. This will allow them to create several different reflections and rotations of the original figure.

To type L_1 , students need to press 2^{nd} [1].

To type L_2 , students need to press 2^{nd} [2].

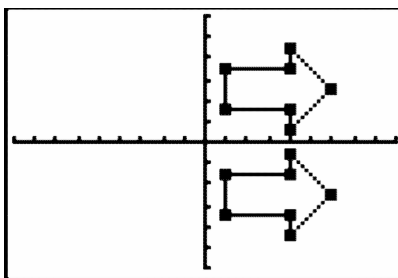
For each combination of lists, students are to determine what type of *reflection* occurred.

A: $x \leftarrow L_3$ and $y \leftarrow L_2$



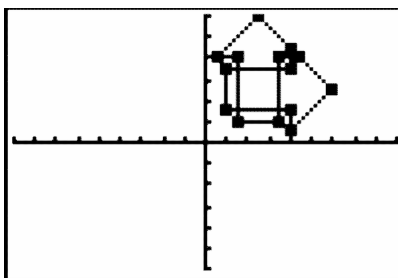
$(-x, y)$ over y -axis

B: $x \leftarrow L_1$ and $y \leftarrow L_4$



$(x, -y)$ over x -axis

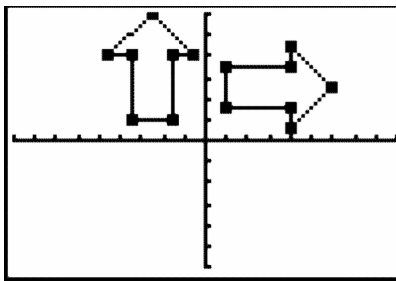
C: $x \leftarrow L_2$ and $y \leftarrow L_1$



(y, x) over the line $y = x$

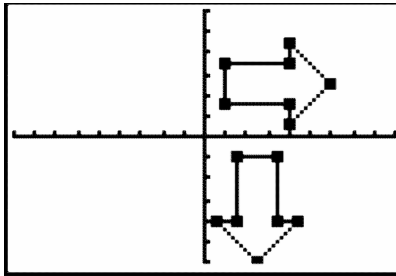
Use **Plot2** to create the following scatter plots. For each combination, determine what type of *rotation* occurred.

D: $x \leftarrow L_4$ and $y \leftarrow L_1$



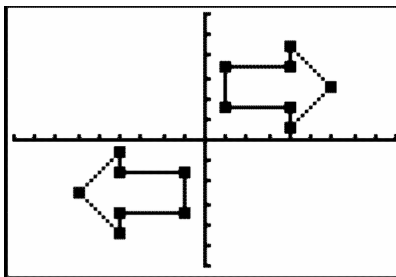
$(-y, x)$ 90° around origin

E: $x \leftarrow L_2$ and $y \leftarrow L_3$



$(-x, -y)$ 90° around origin

F: $x \leftarrow L_3$ and $y \leftarrow L_4$



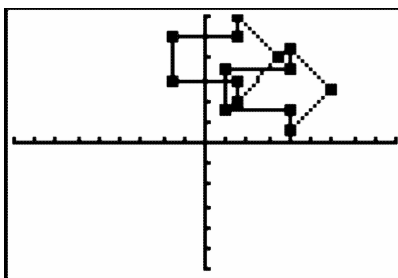
$(y, -x)$ 180° around origin

Problem 3 – Translations

In the list editor, students are to enter the formulas $= L_1 - 5$ and $= L_2 + 3$ for L_3 and L_4 respectively. This will allow them to translate the original figure.

Students should see that the image shifted to the left 5 units and up 3 units. Remind students that the tick marks on the graph are every 2 units.

13.1. TRANSFORMATIONS WITH LISTS

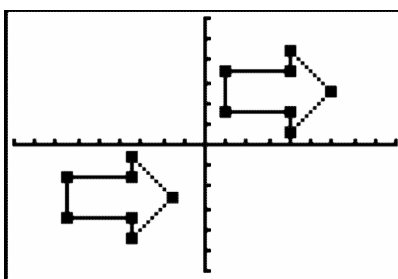


Now students are to translate the scatter plot into Quadrant 3 by editing the formula bars for L_3 and L_4 . Possible formulas are below.

$$L_3 \text{ formula: } = L_1 - 15$$

$$L_4 \text{ formula: } = L_2 - 10$$

The image shifted 15 units to the left and 10 units down.



Extension – Dilations

Remind students that they have seen dilations in Chapter 7, Lesson 6.

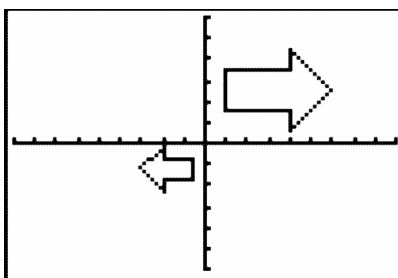
In the list editor, students are to enter the formulas $= 0.5 * L_1$ and $= 0.5 * L_2$ for L_3 and L_4 respectively. This will allow them to dilate the original figure.

Students should see that the image decreased in size. If they have trouble seeing the image, they can the mark of the plot to the small dot.

Then students are to dilate the scatter plot into Quadrant 3 by editing the formula bars for L_3 and L_4 . Remind students that the scale factor needs to be the same for both lists. Possible formulas are below.

$$L_3 \text{ formula: } = -0.5 * L_1$$

$$L_4 \text{ formula: } = -0.5 * L_2$$



13.2 Reflections and Rotations

This activity is intended to supplement Geometry, Chapter 12, Lesson 5.

ID: 8180

Time required: 45 minutes

Activity Overview

Students draw a puppet whose arms and legs are constructed by using a translation vector and reflection line. They will manipulate the vector and observe the changes in the puppet. This activity will help students visualize the effect of a transformation in their construction.

Topic: Transformational Geometry

- *Given a translation vector and a geometric figure, translate the figure to discover that lengths, angles, areas, shapes and parallel line segments are preserved under translations.*
- *Given a reflection line and a geometric figure, reflect the figure to discover that lengths, angles, areas and shapes are preserved under reflections and orientations are reversed.*

Teacher Preparation and Notes

- *Although this is a geometry activity, it can also be used in a Pre-Algebra or Algebra classroom.*
- *Before beginning this activity, students should be familiar with reflections and translations outside the coordinate plane. They should already know the terms associated with transformations, such as line of reflection and translation vector.*
- *This activity is designed to be explored individually or in pairs, with the teacher acting as a facilitator.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*

Associated Materials

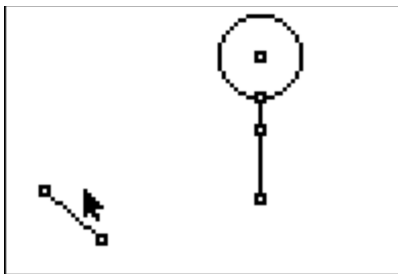
- Student Worksheet: Reflections #38; Rotations <http://www.ck12.org/flexr/chapter/9697>, scroll down to the second activity.
- Cabri Jr. Application

Problem 1 – Constructing the puppet

In *Cabri Jr.*, students will start with a new document.

They will use the **Circle**, **Segment**, and **Point On** tools to create the head, body, and shoulder, respectively.

Students *must* draw the segment for the translation vector in the order described on the worksheet (bottom right point, then top left point).

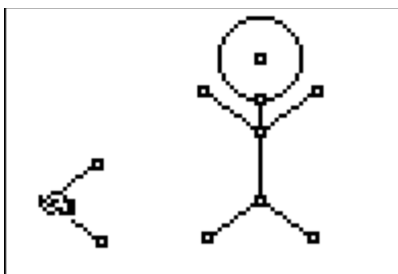


They are directed to use the **Translation** tool to create the left hand and then use the **Reflection** tool to create the right hand. Each point will need to be joined with a segment to the shoulder point.

Another segment to be used as the translation vector for the legs will be drawn with the first one. Again, students *must* draw the segment in the order described on the worksheet (top right point, then bottom left point).

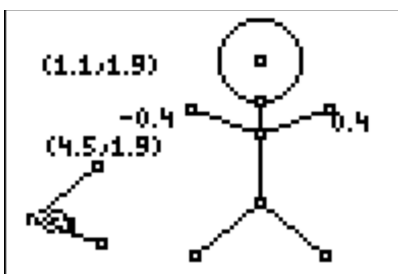
Problem 2– Moving the puppet

Students should drag the point joining the two translation vectors and observe the changes of the arms and legs.



To investigate the puppet further, students should use the **Slope** tool to find the slopes of the arms. Then they can drag the point on the left side of the screen and observe the changes.

Students can also find the coordinates of the left and right hands.



13.3 Perspective Drawing

This activity can be used as an end of the year activity or to supplement the "Know What?" from Chapter 7, Lesson 6.

ID: 10033

Time required: 60 minutes

Activity Overview

In this activity, students draw figures in one- and two-point perspective and compare and contrast the two types of drawings. They then create an isometric drawing and compare it to their drawings in perspective.

Topic: 3–Dimensional Geometry

- *Construct 3- dimensional prisms and pyramids.*
- *Record the number of faces, edges, and vertices of prisms.*

Teacher Preparation and Notes

- *Perspective drawings can be taught at any time in a geometry curriculum, but are most appropriate after lessons on parallel and perpendicular lines, three-dimensional figures, and symmetry.*
- *Throughout the activity, students use many drawing and construction tools, such as the **Segment, Parallel, and Perpendicular** tools. In this document, the first use of a tool is by name and accompanied by its location within the menu structure. For subsequent uses, the tool may be mentioned by name or its function, and the menu location is omitted.*
- *This activity is designed to have students explore **individually or in pairs**.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator files, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=1003> and select **RECPRISM, TRIPRISM, TWOPERSP, ISODRAW**.*

Associated Materials

- Student Worksheet: Perspective Drawing <http://www.ck12.org/flexr/chapter/9697>, scroll down to the third activity.
- Cabri Jr. Application
- RECPRISM.8xv, TRIPRISM.8xv, TWOPERSP.8xv, and ISODRAW.8xv

Problem 1 – One-point perspective

Students will open the *Cabri Jr.* file *RECPRISM* and find a vanishing point A along the horizon and a rectangle $BCDE$.

They are to use the **Segment** tool from the $F2$ menu to draw \overline{AB} and \overline{AC} . These two segments (and any other segment that joins a vertex of the prism to the vanishing point) are hereafter referred to as *vanishing segments*.

Next, they will create \overline{FG} such that F is between A and B , G is between A and C , and $\overline{FG} \times \overline{BC}$ (Refer to the diagram at the right).

To do this, they will need to do the following:

- Place a point F on \overline{AB} . (**F2 #62; Point #62; Point On**)

Construct a line through F parallel to \overline{BC} .

(**F3 #62; Parallel**)

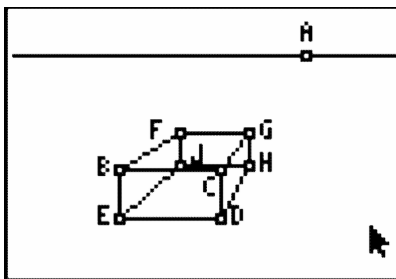
- Plot point G at the intersection of the parallel line and \overline{AC} . (**F2 #62; Point #62; Intersection**)
- Hide the line. (**F5 #62; Hide/Show #62; Object**)
- Draw \overline{FG} .

Students can now hide the vanishing segments, \overline{AB} and \overline{AC} , and draw \overline{BF} and \overline{CG} . The front and top faces of the rectangular prism have been constructed.

Ask students if they are satisfied that this drawing completely represents a rectangular prism. Have them drag point A (press **APLHA** to grab the point) to either the far left or far right and ask students what is wrong with the drawing. (The rectangular prism does not have a face on either side.)

To complete the prism, students next need to construct \overline{AD} and \overline{AE} . Then, using the **Perpendicular** tool (**F3 #62; Perp.**), construct two lines perpendicular to \overline{FG} —one through F and the other through G . The vertical “back” edges \overline{GH} and \overline{FJ} may then be created by plotting the intersection points, hiding the lines, and constructing the segments as needed.

To show all six edges, students should draw a segment connecting H and J . Once all of the vanishing segments are hidden, students will find they need to also construct segments \overline{DH} and \overline{EJ} .



Depending on the topics you have recently covered in class, you can now have students identify parallel, intersecting, and skew lines; count faces, vertices, and edges; or discuss the similarity of rectangles $BCDE$ and $FGHJ$.

When students have finished exploring their prisms, they may wish to hide or dash the edges that would not be visible if the figure were not transparent. The appearance of the edges may be altered using the **Display** tool (**F5 #62; Display**).

Students are to open **TRIPRISM** and create a triangular prism in one-point perspective on their own. Be sure to circulate around the room and assist students as needed.

Allow students a few minutes to explore the figure by dragging point A along the horizon or dragging an edge. Vertices B , C , and D can also be dragged on this figure.

Problem 2 – Two-point perspective

The Cabri Jr. file **TWOPERSP** contains two vanishing points, A and B , signifying this rectangular prism will be drawn in two-point perspective. The file also contains a vertical line segment, \overline{CD} , which will be the front edge of the prism.

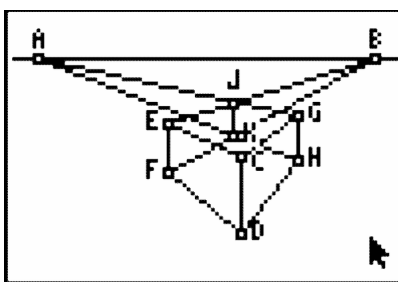
\overline{CD} is constructed so that dragging point C changes the segment's length and dragging point D translates the segment.

Next, students will then use the **Segment** tool to draw the vanishing segments \overline{AC} , \overline{AD} , \overline{BC} , and \overline{BD} . Then they should use the **Point On, Parallel, Intersection, Hide/Show**, and **Segment** tools (as they did in Problem 1) to construct the vertical edges, \overline{EF} and \overline{GH} , of the left and right faces.

The top and bottom edges of those faces, \overline{CE} , \overline{CG} , \overline{DF} , and \overline{DH} may be drawn as well.

Students should now hide the four vanishing segments, and then draw four more: from E and F to B and from G and H to A .

The intersection of the upper two vanishing segments and that of the lower two vanishing segments should be plotted and a segment drawn between them to form the last vertical edge, \overline{JK} .



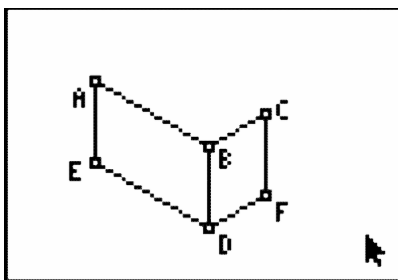
The vanishing segments may then be hidden so that students can draw the remaining edges of the prism.

As before, allow students some time to drag edges, vertices, and vanishing points, observing how the prism changes as they do so. (Dragging \overline{CD} is very interesting!) Again, students can either hide or dash any “hidden edges.”

Problem 3 – An isometric drawing

Using **ISODRAW**, students will construct an isometric drawing of a rectangular prism. The three segments shown represent the three “front” edges of the prism, and each of the angles the segments form measures 120° . Students should first use the **Parallel, Intersection, Hide/Show**, and **Segment** tools to construct \overline{AE} , \overline{CF} , \overline{DE} , and \overline{DF} , as shown in the diagram to the right.

They may then use the same sequence of tools once more to construct the remaining edges: \overline{AG} , \overline{CG} , \overline{EH} , \overline{FH} , and \overline{GH} .



After completing the isometric drawing, students should compare it to the perspective drawings from Problems 1 and 2. As a similarity, students may identify that right angles “in real-life” do not actually measure 90° in the drawings. As a difference, all parallel lines “in real-life” are parallel in an isometric drawing, whereas in perspective drawings, *some* parallel lines actually “meet” at the vanishing point.

